Open Science Journal of Statistics and Application
2019; 6(1): 1-5
http://www.openscienceonline.com/journal/osjsa
ISSN: 2381-4950 (Print); ISSN: 2381-4969 (Online)

A Parametric Technique Based on Simplex for Treating Stochastic Multicriteria Linear Programming Problem

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To cite this article

Adel Mefleh Widyan. A Parametric Technique Based on Simplex for Treating Stochastic Multicriteria Linear Programming Problem. *Open Science Journal of Statistics and Application*. Vol. 6, No. 1, 2019, pp. 1-5.

Received: March 9, 2019; **Accepted**: April 22, 2019; **Published**: May 9, 2019

Abstract

Most of works treats the stochastic multicriteria optimization linear programming problem (SMOLPP) by transforming it to a deterministic one and solve it to obtain the efficient solutions. This paper introduced a new parametric technique based on the simplex algorithm for decomposing the parametric space of SMOLPP. The mathematical expectation approach will be used to transform the stochastic model with random variable in the objective functions to a deterministic one. Then, using the nonnegative weighted sum approach to scalarize the problem. A set of all efficient solutions can be determined using the proposed approach. A numerical example will be given to illustrate the proposed algorithm.

Keywords

Stochastic Optimization, Multicriteria Programming, Parametric Study, Simplex Techniques

1. Introduction

Stochastic multicriteria optimization programming problems (SMOPP) arises in many real life situations with random variables in the objective functions, the constraints or both. Stochastic programming as an optimization technique have been developed by different approaches, chance constrained method has been applied by Charnes and Cooper [1]. Widyan [2] determines the efficient solutions set for Multiobjective Optimization Programming Problem (MOPP) with random variables in both the objective function and the constraints based on the expected value approach. While, J. Fliege and H. Xu [3] used sample average approximation instead of expected value for solving stochastic multiobjective optimization problem.

The simplex techniques is considered one of the most popular approaches in mathematical computation [4]. Yu and Zeleny [5] used a Multiobjective simplex method to derive the set of non-dominated front solution for linear biobjective Programming Problems. Widyan [6] proposed an algorithm to generate a solution set and determine the stability set of the first kind for decomposing the parametric space of SMOPP. N. Brysen [7] presented five applications of the parametric programming procedures. J. A. Filan et al [8] develop asymptotic simplex technique for parametric linear programming.

M. Ehrgott et al [9] proposed an algorithm based on the simplex technique to solve the multiobjective linear programming problems. B. Rudloff et al [10] presented a parametric simplex approach for solving multicriteria linear programming problems. Abo-Sinna and Hussein [11] applied weighted norm method to present an algorithm of decomposition the parametric space of the multicriteria dynamic programming problems. Andrzej [12] reviewed some developmental decomposition techniques for linear multi stage stochastic optimization programming problems.

This work focuses on proposing a new parametric approach using simplex technique for decomposing the parametric space of SMOLPP. In addition, mathematical expectation is used to convert the stochastic problem into a deterministic one. The set of all efficient solutions of SMOLPP is generated using both the nonnegative weighted sum approach and the proposed algorithm.

2. Problem Formulation

The SMOLPP can be formulated as follows:

$$\text{Min}_{x\in M}\ \sum_{j=1}^{n} c_{ij}x_j\,, i=1,2,\dots,m \tag{1}$$

Where

$$M=\left\{x\in R^n / \sum_{j=1}^{n} a_{ij}x_j \le \mathrm{b}, x_j\ge 0, i=1,2,\dots,k\right\}$$

and

$c_{i.}: i=1,2,\dots,m$, are m-dimensional stochastic parameters.

Using mathematical expectation, problem (1) can be transformed to an equivalent deterministic multicriteria optimization problem in the following form [2]:

$$\text{Min}_{x\in M}\ \sum_{j=1}^{n} \mu_{c_{ij}}x_j\,, i=1,2,\dots,m \tag{2}$$

The problem (2) can be converted to a single objective optimization form by applying the non-negative weighted sum approach as follows:

$$\text{Min}_{x\in M}\ \sum_{j=1}^{n} \gamma_i\mu_{c_{ij}}x_j\,, i=1,2,\dots,m \tag{3}$$

Where

$$\sum_{i=1}^{m}\gamma_i=1\,,\ \gamma_i\ge 0, i=1,2,3,\dots,m$$

Definition 2.1: "A point $\bar{x}\in M$ is said to be an efficient solution of $(P1)$ if there is no $x\in M$ such that:

$$\sum_{j=1}^{n}\mu_{c_{ij}}\bar{x}_j \le \sum_{j=1}^{n}\mu_{c_{ij}}x_j\ \,, i=1,2,\dots,m \tag{4}$$

With

$$\sum_{j=1}^{n}\mu_{c_{ij}}\bar{x}_j < \sum_{j=1}^{n}\mu_{c_{ij}}x_j$$

For at least one $i\in\{1,2,\dots,m\}$". [13].

Corollary 2.1: "If the problem (1) has a bounded feasible set, then it has efficient solutions." [4]

3. Multicriteria Optimization Simplex Technique

Here, this paper interested with optimization linear programming problem involving multicriteria functions. Many researches concentrated on expansions of the simplex technique and algorithm to treat the bicriteria. However, simplex technique have established to be highly efficient for various categories of one objective linear programming problems.

By applying the simplex technique, firstly, construct the initial simplex tableau, which takes the following form:

Table 1. *The Simplex Tableau Form.*

B	***N***	***b***
C_B	C_N	$0_{m\times 1}$

Where; $[C_B\ C_N]$ the coefficient matrices of the objective functions corresponding to $[\ B\ N\ ]$ basic and nonbasic matrices represent the left hand side coefficients of the constraints and b is a positive vector of right hand side of the constraints, 0 is a vector of slackness coefficients.

To examine the efficiency of the point $\bar{x}$, compute

$$\bar{b}=B^{-1}b$$

$$\bar{C}_N=C_N-C_BB^{-1}N \tag{5}$$

Where, $\bar{C}_N$ represents the reduced cost matrix at the basis B. [4]

Definition 3.1: "A feasible basis B is called efficient basis if B is an optimal basis of $LP(\gamma)$ for some $\gamma\in R^m$." [13]

Definition 3.2: "Two bases B and $\bar{B}$ are called adjacent if one can be obtained from the other by a single pivot step." [13]

Definition 3.3: "Let B be an efficient basis. Variable $x_j, j\in N$ is called efficient nonbasic variable at B if there exists a $\gamma\in R^m$ such that $\gamma^TC\ge 0$ and $\gamma^Tr_j=0$, where r_j is the column of C corresponding to variable x_j." [13]

Definition 3.4: "Let B be an efficient basis and let x_j be an efficient nonbasic variable. Then a feasible pivot from B with x_j entering the basis is called an efficient pivot with respect to B and x_j." [13]

Proposition 3.1: "Let B be an efficient basis. There exists an efficient nonbasic variable at B." [13]

4. Stochastic Multicriteria Optimization Simplex Algorithm

4.1. Flowchart of Simplex Technique

Figure 1 describes the procedure of proposed technique.

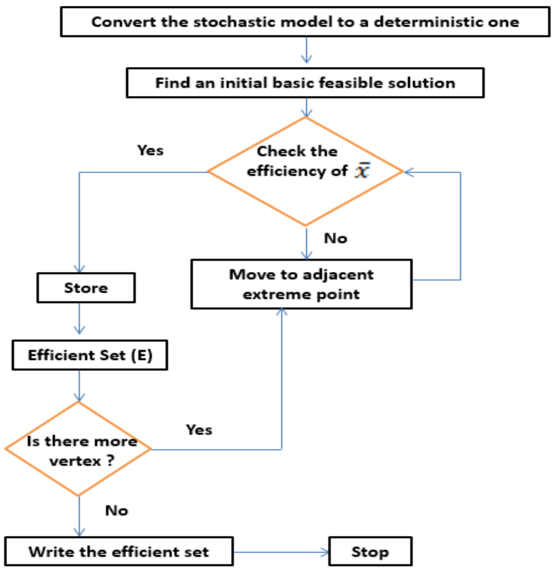

Figure 1. *Flowchart of the Proposed Algorithm.*

Theorem 4.1: "If all feasible bases of the matrix A are non-degenerate, then the simplex algorithm terminates at a finite number of iterations." [4]

4.2. Algorithm

The following simplex algorithm is constructed to generate the efficient solution and store them in E.

Step 1. Covert the stochastic optimization Programming model to a deterministic one.

Step 2. Using the Two-Phases simplex algorithm to find an initial basic feasible solution of the problem (3) (Phase I), say $\bar{x}$.

Step 3. If the problem (3) is infeasible then, stop. Otherwise, go to step 4.

Step 4. Construct the initial simplex tableau (Table 1).

Step 5. Examine the efficiency of $\bar{x}$:

1. If $\bar{C}_N \leq 0$, stop (the ideal solution is an efficient)
2. Otherwise, go to step 6.

Step 6. Examine if any row of $\bar{C}_N < 0$, or the rows sum of $\bar{C}_N$ is negative or not:

1. If yes, the solution is efficient, go to step 8.
2. Otherwise, go to step 7.

Step 7. Get an efficient solution dominating $\bar{x}$ by solving the problem (3) (phase-II) which stated as follows:

$$\mathrm{Min}_{x \in M} \sum_{j=1}^{n} \gamma_i \mu_{c_{ij}} x_j \,, i = 1, 2, \dots, m \tag{6}$$

Subject to

$$C_x \geq C_{x^i}$$

1. If problem (6) has unbounded solution then stop.
2. If problem (6) has bounded solution at x^i, then x^i is an efficient solution and go to step 8.
3. If $\bar{x}$ is dominate x^i, then x^i is not efficient, therefore, we construct a new simplex tableau of $\bar{x}$.

Step 8. Store of x^i as efficient solution in E.

Step 9. Get all non-basic indices r of the last stored vertex, which contains mixed components of $\bar{C}_r$ (positive and negative ones), go to step 10, otherwise stop.

Step 10. Let $i=i+1$ and the new vertex is x^{i+1}, go to step 4.

5. Numerical Example

Consider the following numerical example to illustrate the proposed algorithm. The problem is consider to be multicriteria optimization linear programming with random parameter in the objectives.

$$\mathrm{Min}_{x \in M} \left\{ \sum_{j=1}^{3} c_{ij} x_j \,, i = 1, 2, 3 \right\} \tag{7}$$

Where, $c_{ij}, i = 1, 2, 3, j = 1, 2, 3$ are random variables with expected values as follows:

$$\mu_{c_{11}} = 3, \mu_{c_{12}} = \frac{5}{2}, \mu_{c_{13}} = \frac{3}{2}$$

$$\mu_{c_{21}} = \frac{3}{2}, \mu_{c_{22}} = 3, \mu_{c_{23}} = \frac{5}{2}$$

$$\mu_{c_{31}} = \frac{5}{2}, \mu_{c_{32}} = 2, \mu_{c_{33}} = 4,$$

and

$$M = \left\{ x \in R^3 \left| \begin{array}{c} x_1 + x_2 + \frac{1}{2} x_3 \leq 3 \\ x_1 + 2x_2 + 5x_3 \leq 6 \\ \frac{1}{2} x_1 + 2x_2 + \frac{1}{2} x_3 \leq 4 \\ x_1, x_2, x_3 \geq 0 \end{array} \right. \right\} \tag{8}$$

After using mathematical expectation to transform the stochastic model into a deterministic model and the problem will be written in the standard form by adding three slack variables s_1, s_2 and s_3.

$$Min \begin{pmatrix} 2 & \frac{5}{2} & \frac{3}{2} & 0 & 0 & 0 \\ \frac{3}{2} & 3 & \frac{5}{2} & 0 & 0 & 0 \\ \frac{5}{2} & 2 & 4 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix} \tag{9}$$

Subject to

$$\begin{pmatrix} 1 & 1 & \frac{1}{2} & 1 & 0 & 0 \\ 1 & 2 & 5 & 0 & 1 & 0 \\ \frac{1}{2} & 2 & \frac{1}{2} & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ 4 \end{pmatrix} \tag{10}$$

$$x_1, x_2, x_3 \geq 0 \tag{11}$$

The initial simplex tableau is given as bellow:

Table 2. *The Initial Simplex Tableau.*

		x_1	x_2	x_3	s_1	s_2	s_3	b
	s_1	1	1	$\frac{1}{2}$	1	0	0	3
	s_2	1	2	5	0	1	0	6
	s_3	$\frac{1}{2}$	1	$\frac{1}{2}$	0	0	1	4
$T^0 =$	f_1	-2	$-\frac{5}{2}$	$-\frac{3}{2}$	0	0	0	0
	f_2	$-\frac{3}{2}$	-3	$-\frac{5}{2}$	0	0	0	0
	f_3	$-\frac{5}{2}$	-2	-4	0	0	0	0
	Sum	-6	$-\frac{15}{2}$	-8	0	0	0	0

The basic solution is $x^0 = (0, 0, 0, 3, 6, 4)$.

Table 3. *The First Simplex Tableau.*

		x_1	x_2	x_3	s_1	s_2	s_3	b
	s_1	$\frac{9}{10}$	$\frac{4}{5}$	0	1	$-\frac{1}{10}$	0	$\frac{12}{5}$
	x_3	$\frac{1}{5}$	$\frac{2}{5}$	1	0	$\frac{1}{5}$	0	$\frac{6}{5}$
	s_3	$\frac{2}{5}$	$\frac{9}{5}$	0	0	$-\frac{1}{10}$	1	$\frac{17}{5}$
$T^1 =$	f_1	$-\frac{17}{10}$	$-\frac{19}{10}$	0	0	$\frac{3}{10}$	0	$\frac{9}{5}$
	f_2	-1	-2	0	0	$\frac{1}{2}$	0	3
	f_3	$-\frac{17}{10}$	$-\frac{2}{5}$	0	0	$\frac{4}{5}$	0	$\frac{24}{5}$
	Sum	$-\frac{22}{5}$	$-\frac{43}{10}$	0	0	$\frac{8}{5}$	0	$\frac{48}{5}$

The new basic solution is $x^1 = \left(0, 0, \frac{6}{5}, \frac{12}{5}, 0, \frac{17}{5}\right)$.

Table 4. *The Second Simplex Tableau.*

		x_1	x_2	x_3	s_1	s_2	s_3	b
$T^2 =$	x_1	1	$\frac{8}{9}$	0	$\frac{10}{9}$	$-\frac{1}{9}$	0	$\frac{8}{3}$
	x_3	0	$\frac{2}{9}$	1	$-\frac{2}{9}$	$\frac{2}{9}$	0	$\frac{2}{3}$
	s_3	0	$\frac{13}{9}$	0	$-\frac{4}{9}$	$-\frac{1}{18}$	1	$\frac{7}{3}$
	f_1	0	$-\frac{7}{18}$	0	$\frac{17}{9}$	$\frac{1}{9}$	0	$\frac{19}{3}$
	f_2	0	$-\frac{10}{9}$	0	$\frac{10}{9}$	$\frac{7}{18}$	0	$\frac{17}{3}$
	f_3	0	$\frac{10}{9}$	0	$\frac{17}{9}$	$\frac{11}{18}$	0	$\frac{28}{3}$
	Sum	0	$-\frac{17}{18}$	0	$\frac{44}{9}$	$\frac{10}{9}$	0	$\frac{64}{3}$

The new basic solution is $x^2 = \left(\frac{8}{3}, 0, \frac{2}{3}, 0, 0, \frac{7}{3}\right)$.

Table 5. *The Third Simplex Tableau.*

		x_1	x_2	x_3	s_1	s_2	s_3	b
$T^3 =$	x_1	1	0	0	$\frac{18}{13}$	$-\frac{1}{13}$	$-\frac{8}{13}$	$\frac{16}{13}$
	x_3	0	0	1	$-\frac{2}{13}$	$\frac{3}{13}$	$-\frac{2}{13}$	$\frac{4}{13}$
	x_2	0	1	0	$-\frac{4}{13}$	$-\frac{1}{26}$	$\frac{9}{13}$	$\frac{21}{13}$
	f_1	0	0	0	$\frac{23}{13}$	$\frac{5}{52}$	$\frac{7}{26}$	$\frac{181}{26}$
	f_2	0	0	0	$\frac{10}{13}$	$\frac{9}{26}$	$\frac{10}{13}$	$\frac{97}{13}$
	f_3	0	0	0	$\frac{29}{13}$	$\frac{17}{26}$	$-\frac{10}{13}$	$\frac{98}{13}$
	Sum	0	0	0	$\frac{62}{13}$	$\frac{57}{52}$	$\frac{7}{26}$	$\frac{571}{26}$

The new basic solution is $x^3 = \left(\frac{16}{13}, \frac{21}{13}, \frac{4}{13}, 0, 0, 0\right)$.

From the objective rows of nonzero columns, we can get $P(\gamma)$ to be the solution set of the following system:

$$(\gamma_1, \gamma_2, \gamma_3)\begin{bmatrix} \frac{23}{13} & \frac{5}{52} & \frac{7}{26} \\ \frac{10}{13} & \frac{9}{26} & \frac{10}{13} \\ \frac{29}{13} & \frac{17}{26} & \frac{-10}{13} \end{bmatrix} \geq 0 \tag{12}$$

$$\frac{23}{13}\gamma_1 + \frac{10}{13}\gamma_2 + \frac{29}{13}\gamma_3 \geq 0 \tag{13}$$

$$\frac{5}{52}\gamma_1 + \frac{9}{26}\gamma_2 + \frac{17}{26}\gamma_3 \geq 0 \tag{14}$$

$$\frac{7}{26}\gamma_1 + \frac{10}{13}\gamma_2 - \frac{10}{13}\gamma_3 \geq 0 \tag{15}$$

Let $\gamma_3 = 1 - \gamma_1 - \gamma_2$

(13), (14) and (15) become,

$$\frac{6}{13}\gamma_1 + \frac{19}{13}\gamma_2 \leq \frac{29}{13} \text{ then, } \gamma_2 \leq \frac{29}{19} - \frac{6}{19}\gamma_1 \tag{16}$$

$$\frac{29}{52}\gamma_1 + \frac{4}{13}\gamma_2 \leq \frac{17}{26} \text{ then, } \gamma_2 \leq \frac{17}{8} - \frac{29}{16}\gamma_1 \tag{17}$$

$$\frac{27}{26}\gamma_1 + \frac{20}{13}\gamma_2 \geq \frac{10}{13} \text{ then, } \gamma_2 \geq \frac{1}{2} - \frac{27}{40}\gamma_1 \tag{18}$$

To reduce the size of the dimension of the graphic representation, we have noticed that each of $P(x^3)$ is one-to-one corresponding to a vertex of the simplex:

$$S = \{(\gamma_1, \gamma_2, \gamma_3): \gamma_1 + \gamma_2 + \gamma_3 = 1, \gamma_1, \gamma_2, \gamma_3 \geq 0\} \tag{19}$$

Then, let $\gamma_3 = 1 - \gamma_1 - \gamma_2$ and the redundant constraints of (12) will be ignored, then we get the parametric set in S at $x^3 = \left(\frac{16}{13}, \frac{21}{13}, \frac{4}{13}, 0, 0, 0\right)$ as:

$$P(x^3) = \{(\gamma_1, \gamma_2): \gamma_2 \leq 1 - \gamma_1, 27\gamma_1 + 40\gamma_2 \geq 20, \gamma_1, \gamma_2 \geq 0\} \tag{20}$$

It is noticed that $(P(x^3) \cap S) \neq \emptyset$, therefore, the set of $(P(x^3) \cap S)$ is depicted in the following figure.

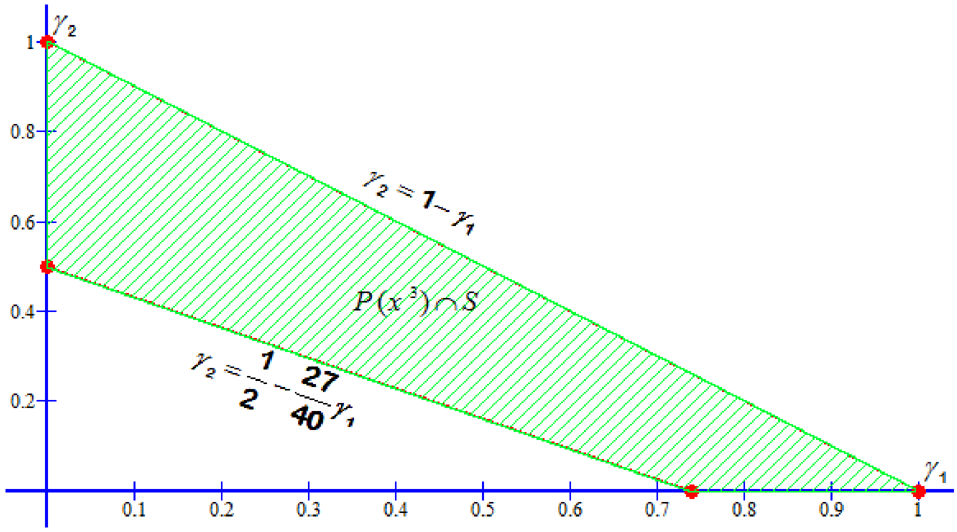

Figure 2. *The Parametric Space.*

6. Conclusion

This paper used the mathematical expectation method to transform the stochastic model into a deterministic one. A new parametric approach is proposed for generating the set of all efficient solution of SMOLPP based on the simplex technique. Also, the decomposition of the parametric space through the nonnegative weighted sum approach is introduced. The validity of the proposed algorithm has been verified by applying it on a numerical example involves stochastic parameters in the objective functions.

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