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Cooperative Learning Strategy in Teaching and Learning of Circle Theorem in Mathematics: A Case of Nana Brentu Senior High School in Aowin Municipality in Ghana

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Abstract

Knowing the factors affecting math achievement is particularly important for making the best design decisions. The study investigated the impact of cooperative learning strategy on mathematics achievements of students. The study adopted action research design and made use of 60 students (42 boys and 18 girls) from Brentu Senior High Technical School. Purposive sampling technique was used to select Form 2 Business class out of seven Form 2 classes. The main instruments used for data collection were pre-test, post-test and questionnaires. Data collected were analysed and presented using frequency tables, pictorial representations in the form of bar graph, descriptive and inferential approaches of statistics. The Two-tailed test was used in the inferential statistics to test the null hypothesis. It was found that students' performance in mathematics and attitudes towards mathematics were influenced by exposure to the cooperative learning (Pre-test: Mean = 35.20, SD =20.535; Post-test: Mean = 57.27, SD =17.829). The results further showed that there was significant difference in the performance of students when they are taught Circle Theorem using cooperative instructional strategies ($t = 3.157$, $p = 0.003$). It was therefore, recommended that mathematics teachers should employ cooperative learning strategies to improve students' performance to bridge the gap among high, medium and low achievers.

Keywords

Cooperative Learning, Strategy, Teaching and Learning, Circle Theorem, Mathematics, Nana Brentu, Senior High School, Aowin Municipality, Ghana

1. Introduction and Background

This paper presents the outcome of the cooperative learning strategy in teaching and learning of circle theorem in mathematics Senior High Schools in Ghana. The paper is structured into five (5) main sections namely; the Introduction, Review of the Literature, Methodology, Findings and Discussion and the Conclusion. The first section introduces the structure of the paper, the context and

aims and objectives of the paper. The literature review section reviews the relevant literature on issues pertaining to the impact of cooperative learning strategy in teaching and learning of circle theorem in mathematics. The methodology section presents a broad description of the methodology and procedures adopted in the conduct of the study. Findings resulting from the study are presented and discussed in the section following the methodology and conclusions with recommendations.

Students' mathematical achievements in senior high school have an influential effect on their performance in college and their future careers. Having a solid background in mathematics helps students develop sophisticated perspectives and offers more career options. The importance of mathematical learning has repeatedly been emphasized by educators and politicians [32]. Both teachers and parents have paid attention to students' performance in mathematics and their progress every year.

The quality of teaching and learning of mathematics has been one of the major challenges and concerns of educators. Education is to assist individuals to maximize their potentials for optimal self and national development. Education is a prerequisite for meaningful and sustained national economy. Education cannot be of quality without effective teaching. The instructional method employed by teacher plays an important role in the acquisition of instructional contents for meaningful learning and development of necessary skills. Teacher-centred instructional methods make students passive with less interaction. Lack of active participation of students is one of the factors responsible for students' poor performances in mathematics [17]. Senior high schools' students' performances are generally poor. In Africa the factors attributed to learners' poor performance in mathematics includes: inadequate teaching and learning resources; negative teacher/learner attitude towards the subject; and ineffective teaching methods [18]. Factors that contribute to poor performance in Africa in general and Ghana in particular are similar. There is therefore need for teaching strategies that arouse students' interest to learn mathematics and hence improve the quality of outcomes in mathematics classrooms. However, the issue of poor performance in mathematics examinations was due to problem of teaching methods [2]. There has also been an increasing awareness by those concerned with mathematics education that the conventional methods of teaching mathematics, has not been very successful. For effective teaching to take place, the skilful mathematics teacher needs to use many different methods and techniques at his disposal. A carefully designed teaching method can make teaching and learning effective [7]. One of the many teaching methods proposed results in positive effect on students' achievement and retention of information is cooperative learning [12, 24].

Cooperative learning has been well documented in the educational research as a successful pedagogy to improve students' academic achievement. It is a fundamental principle of cooperative learning that group members are linked together in such a way that they cannot succeed unless everyone succeed, they will actively assist each other to make sure that the assignment is done and the purpose of the group achieved [9]. They acquire this by providing help and cooperation to each other, sharing resources, and encouraging each other's efforts. As a result, group members who work in cooperative groups outperform students who work by themselves or in competition with each other (as seen in competitive conventional classrooms) [14]. Cooperative learning is one of the two ways of organizing the learning environment of a classroom, the other being competitive. In cooperative learning environment, the goals of separate individuals become so linked that there is a positive correlation between them; on the contrary, in a competitive conventional environment, the goals of the students are so linked that there is a negative correlation between their goal attainments [13]. Cooperative learning establishes a community in which students can get help and support from other group members immediately in a non-competitive learning environment, just raising their hands and waiting for the right answers to be given.

The cooperative learning strategy can be used to learn most of the topics in Ghana's Senior High School mathematics syllabus. The effect of the strategy on learners` achievement in the mathematics topic, Geometry and specifically, Circle Theorems was studied. This is one of the major topics in the senior high school mathematics curriculum taught to form two students. It has also been among the difficult topics for students to learn in the senior high school mathematics syllabus in Ghana, in which learners obtain low scores in national examination, and hence the need for the study. Unfortunately, the circle theorem is commonly taught in traditional ways in many SHS. Student's participation in the class consists of simply answering textbook questions. The theorem or rule is provided by the teacher and the student absorbs the information. Students are not given the opportunity to discover and explore theorem or properties on their own. They are merely told about them and are asked, in many cases to memorise them. Circle Theorems are a notoriously troublesome topic for many Senior High School students and even for some teachers. Many students turn out to be very miserable and inattentive in a mathematics class after being taught a topic and discover they could not memorize or recall such a concept with ease, especially in Circle Theorems.

The Chief Examiner`s reports of the West African Examinations Council, on candidates' achievements in the West African Senior School Certificate Examinations (WASSCE) and the Senior high school Certificate Examinations (SSSCE) categorically outlined candidates' weaknesses and difficulties in solving problems on circle theorems [28-30]. These reports further indicated that most candidates avoid problems involving circle theorems. The few candidates that attempt questions on circle theorems do not answer them satisfactorily. The reports did not only emphasize students' weaknesses in circle theorems but also criticized mathematics teachers for contributing to the students' weak achievements in circle theorems. The Chief

Examiners` reports contend that students` inability to perform well in that aspect of mathematics is due to total neglect of that area of mathematics by Senior High School mathematics teachers or to the inability of mathematics teachers to use appropriate teaching methods in handling the topics on Circle Theorems. The reason for this difficulty may vary but this could sometimes be related to the teaching method being used to explain such topics.

It is an undeniable fact that students' express difficulty in understanding and retaining concepts being taught to them by their teachers. How well students retain taught mathematics concept can be traced back to the teaching approach used. To make geometry class, for that matter, circle theorem, more meaningful and exciting for students, the instruction must become more student-centred. Students need to play a more active part in the instruction of circle theorem. They need to discover, on their own, theorems or properties and have time to explore and reflect these new ideas. Much cognitive and psychology mathematics education research indicates that learning occurs as students construct their own meanings [22]. The significance of Mathematics in producing versatile and resourceful graduates that are needed for economic development cannot be over-emphasized. This is why [23] affirmed that Mathematics is a fundamental science that is necessary for understanding of most other fields in education. He stressed further that, it is glaring that no other subject forms such a strong force among the various branches of science. It therefore means that, the position of Mathematics in senior high school curriculum in Ghana is important for scientific development. However, it is disheartening that research and data from National Examination Bodies like West African Examinations Council (WAEC) have shown a consistent poor performance in this subject. That is, despite the usefulness attached to mathematics for an individual and society, learners' achievement in the subject at the end of primary and senior high school national examinations has remained low nationwide. Among many factors contributing to learners' low achievement are the ineffective teaching methods used in mathematics classrooms.

Cooperative learning provides an ideal environment for student exploration and discovery. In an attempt to seek a teaching strategy to improve learners` mathematics achievement, the researchers had found that many students struggle with various concepts in circle theorem. Form 2 Business students seem to experience difficulty understanding and visualizing various properties in the circle theorems. The main purpose of this study was to investigate the impact of cooperative learning strategy on mathematics achievements of students in Brentu Senior High Technical School, Enchi. The study was guided by this research question- To what extent would the use of cooperative learning strategy help students' understanding of circle theorem? and Null hypothesis was formulated and tested at the 0.05 significance level in the study: H_a: There is no significant difference between the mean scores of students' performance, when students are taught Circle Theorem using cooperative instructional strategies. H_a: Null hypothesis: There is significant difference between the mean scores of students' performance, when students are taught Circle Theorem using cooperative instructional strategies.

2. Review of the Literature

Cooperative learning is a kind of learning strategy in which students study together and complete common goals [33]. It can also be defined as "small groups of learners working together as a team to solve a problem, complete a task, or accomplish a common goal" [3]. Basically, the goal of mathematics education is to ensure that all students possess a suitable and sufficient mathematics background to become productive citizens in a society that is characterised by complex information and technology [12]. They further revealed that there is considerable evidence indicating that the goals of mathematics instruction will be better achieved when cooperative learning procedures and strategies are employed. The use of cooperative learning will result in students being more cognitively active, more successful in problem solving, more confident in their maths abilities, less anxious about learning maths, more motivated to take further maths courses, and better able to transfer what they know about maths to career situations.

In a similar study presented a rationale, "why does cooperative learning deserve a central place in mathematics instruction?" The study of mathematics is often viewed as an isolated, individualistic, or competitive matter [20]. Perhaps it is not surprising that many students and adults are afraid of mathematics and develop maths avoidance or maths anxiety. They often believe that only a few talented individuals can function successfully in the mathematical realm. Small – group cooperative learning addresses these problems in several ways. Effectiveness of mathematics integrated learning curriculum within the context of cooperative learning motivate students and teachers to involve themselves in the learning activities [6]. In addition, a study on the impact of cooperative learning on the attitude, confidence, and performance of students in undergraduate discrete mathematics courses indicated that cooperative learning group performed better to significant degree and there was a significant increase in attitude and confidence of students to learn mathematics [21]. A similar study was carried on the effects of cooperative learning on the achievement and attitude toward mathematics of a group of 5^{th} grade students from the United States (Bermuda). Students participated for 12 weeks in Student Team Achievement Division method of cooperative learning in mathematics during the fall semester. The analysis of the pre-test scores and post-test scores revealed positive change in attitude and achievement [27]. Moreover, the reasons why teachers put students in cooperative learning groups are that all students can achieve higher academic success individually than were they to study alone [25]. Consequently, each student must be held individually responsible and accountable for doing his or her own share of the work and for learning what has been targeted to be learned. Notably,

groups cannot function effectively if students do not have and use the needed social skills such as leadership, decision-making, trust-building, communication, and conflict-management skills. For the cooperative learning environment to be successful teachers should teach these skills as purposefully and precisely as academic skills and the learner should utilise the skills they have learnt in completing assigned activities [15, 25, 16].

Notwithstanding, students are placed in groups and expected to use appropriate social and group skills does not mean students will automatically use these skills [25]. To work together as a group, students need to engage in such interactive abilities as leadership, trust-building, conflict-management, constructive criticism, encouragement, compromise, negotiation, and clarifying. Teachers may need to describe the expected social interaction behaviours and attitudes of students and to assign particular students specific roles to ensure that they consciously work on these behaviours in their groups". Cooperative learning is a viable but underused teaching-learning tool as opined [19]. They contend that educators can best utilise this teaching strategy in their classrooms more effectively if they themselves were active participants in their teacher education training programme. They asked these three (3) questions.

1. Can we change the post-secondary instructional paradigm from predominantly lecturers to a student participatory teaching and learning style such as cooperative learning?
2. How do teacher education students internalise cooperative learning techniques into their cognitive domain, so that they can use the techniques with their future students?
3. In our merit based American society marked indelibly with the ideals of individualism and competition to favour competition and collaboration?

There have been surveys conducted in Third World cities such as Nigeria to assess student views of cooperative learning strategies. A similar study was conducted on the attitudes of physics students towards the use of cooperative, competitive and individualistic learning strategies in Nigerian Senior High School. The research design for this study was quasi-experimental. There were a total of one-hundred and forty (140) students taking part in the study who were selected by a random sampling technique. A structured questionnaire titled Students' Attitude Towards Mathematics Questionnaire (SATMQ) on 4-point scale was used to collect the data. Poor student performance can be attributed to poor teaching methods, unqualified and inexperienced teachers; poor student attitude toward mathematics, poor learning environment and gender effect [1]. Also, in the present Nigerian educational system, competition is valued over cooperative learning strategies [1].

The findings showed that cooperative learning strategy was the most effective in facilitating students' attitude towards mathematics. This was then followed by competitive strategies with the individualistic learning strategies being seen to be the least facilitative [1]. Cooperative learning leads to more positive attitudes in several areas. Cooperative learning promoted increased motivation, feelings of personal importance and control, acceptance of heterogeneity and conflicts in groups, and better attitudes towards the teacher. They also discovered that students began selecting group members based upon previous, positive collaborative experiences with them [10]. kids. "Each person tends to focus on gratifying his or her own ends without concern for others [8]. Physical, psychological and material self-indulgence has become a primary concern" (p. 58). As a result, today's youth do not feel connected to the hardships of other individuals. Yet, self-fulfillment does not develop from operating in isolation. True meaning and purpose come from contributing to the welfare of others.

3. Methodology

The study adopted action research design. The population of the study comprised of all SHS 2 Business students of Brentu Senior High Technical School in Enchi in the Western region of Ghana. Purposive sampling technique was used to select 60 students (42 boys and 18 girls) from Form 2 Business class out of seven Form 2 classes for the study. The main instruments for data collections were pre-test, post-test and questionnaire. The data were organized and presented using both descriptive and inferential approaches of statistics. In the descriptive analysis, a simple procedure of percentage comparisons was used together with pictorial representations in the form of bar graph and frequency tables. The Two-tailed test was used in the inferential statistics to test the null hypothesis.

The researchers began the data collection by administering a pre-test to the participants (sample). After the pre-test, the researchers divided the students into 12 groups comprising of 5 members to a group. This was due to the fact that the researchers intended to apply cooperative learning strategy to carry out the intervention after the pre-test. At the end of the intervention, a post-test was also administered under the supervision of the researchers. The time allocation for the post-test was 40 minutes, at the end of which the scripts were collected. All students took part in both the pre-test and post-test. The questionnaire was then administered personally to the students to be answered, a day after the post-test. The questionnaire was administered personally to help improve the collection and response rate of the questionnaire. The questionnaire was collected as soon as it was completed by the respondents. This enabled the researchers to obtain 100% response rate.

The intervention lasted for three weeks. The first two weeks were for the intervention activities, which were specifically based on the scope of the topic as stipulated in the Senior High School mathematics syllabus. The duration for each lesson was 70 minutes. The third week was then allocated for group presentations on the circle theorems. Assignments were also given after each lesson.

Lesson introduction

We began the lesson by going through the definition of a

circle and the terminologies used for circles. The researchers posed a question on the definition of a circle to the class and the students thought about their response. Then students paired with their partner to talk over their ideas. Finally, students shared their ideas with the class.

Observations: The researchers observed that almost all the groups came out with similar answers; 'A circle is a plane figure bounded by a circumference, such that all points on the circumference are equidistant from a fixed point within it'. It was also observed that the students were able to discuss amongst themselves the parts of a circle.

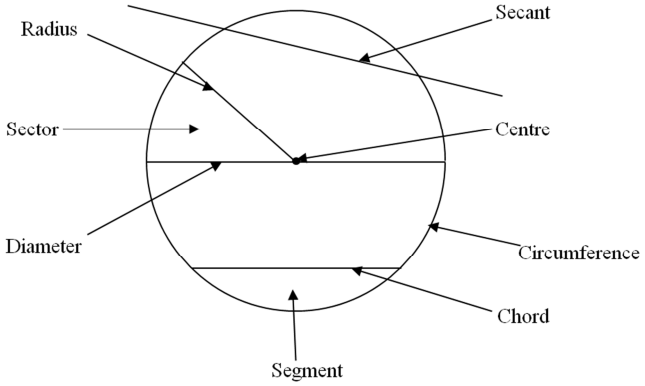

Figure 1. *Parts of a Circle.*

1. A circumference is the total distance around a circle.
2. Any interval joining a point on the circle to the centre is called a radius. By the definition of a circle, any two radii have the same length. Notice that the word 'radius' is being used to refer both to these intervals and to the common length of these intervals.
3. An interval joining two points on the circle is called a chord.
4. A chord that passes through the centre is called a diameter. Since a diameter consists of two radii joined at their endpoints, every diameter has length equal to twice the radius. The word 'diameter' is use to refer both to these intervals and to their common length.
5. A line that cuts a circle at two distinct points is called a secant. Thus a chord is the interval that the circle cuts off a secant, and a diameter is the interval cut off by a secant passing through the centre of a circle centre.

Activity one: The angle subtended by a diameter at the circumference of a circle is a right angle.

Students within the team numbered off from 1-5. The researchers posed a question and the students put their heads together to discuss the answer. The researchers randomly called a number and from each team the student with that number wrote the answer in the team response book.

Observations:

The following observations were made by the students:

The students discussed that sum of interior angles in any triangle is 180°. So by comparison, they noticed that the three angles in the big triangle are a, b and $a + b$.

An equation was then set up:

$2x + 2y = 180$

$x + y = 90$

$x + y$ is therefore a right angle; hence, there was a conclusion that an angle subtended by a diameter at the circumference is a right angle (90°).

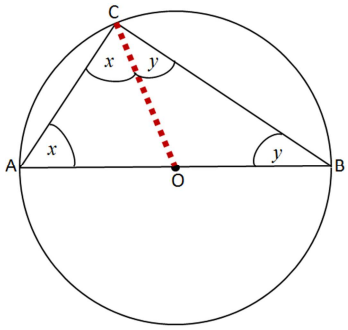

Figure 2. *Equilateral and isosceles triangles.*

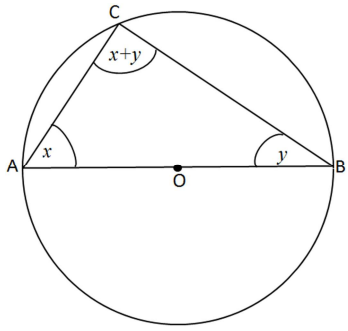

Figure 3. *An angle subtended by a diameter (x+y).*

Activity two: The angle which an arc of a circle subtends at the centre is twice that which it subtends at any point on the circumference.

Students paired within the group and assisted each other to choose two points on the circumference below the centre and one point on the circumference above the centre. They then drew a line connecting each point below the centre to the centre itself and to the point on the circumference above the centre and labelled the angle at the centre and the angle at the circumference. The students further discussed how they could draw a line from the centre to the point on the circumference above the centre in order to obtain two equal isosceles triangles. The researchers were around to monitor the progress of the groups.

Observations:

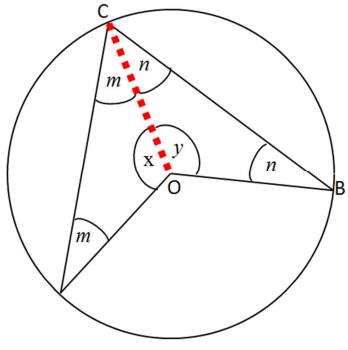

Figure 4. *Two equal isosceles triangles.*

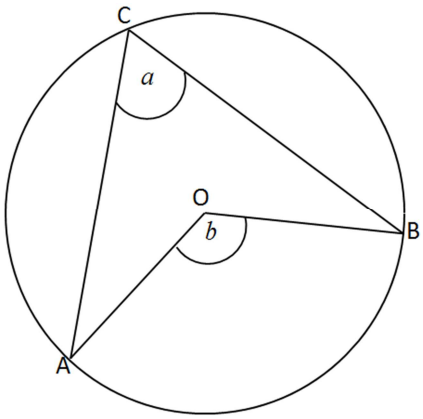

Figure 5. *An angle subtended by an arc.*

The students made the following observations with assistance from the researchers:

The sum of angles inside any triangle is 180°.

Therefore, $2m + x = 180°$ and $2n + y = 180°$.

Thus, $x = 180° - 2m$ and $y = 180° - 2n$.

Angles round a point add up to 180°, so $b + x + y = 360°$.

Therefore $(180° - 2m) + (180° - 2n) + b = 360°$.

Consequently $360° - 2m - 2n + b = 360°$.

So $y - 2m - 2n = 0$.

As a result, $b = 2m + 2n$, therefore $b = 2(m + n)$.

Using the fact that $m + n = a$, there was a conclusion that $b = 2a$.

Activity three: Angles in the same segment of a circle are equal.

A member of each group led and discussed amongst themselves how to draw a circle, marked its centre and put a chord inside. Chord forms two segments. They further planned how to draw two triangles that share the chord as one of their sides. The students then labelled the angles opposite the chord in each triangle. They again decided on how to draw a third triangle – starting at the points where the chord touches the circumference, but with the other vertex at the centre of the circle. Students worked while the researchers went round in the class and watched the social skills, level of cooperation, level of interaction and level of participation.

Observations:

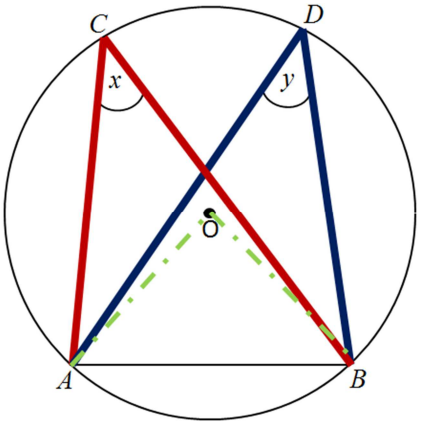

Figure 6. *Segments formed by chord.*

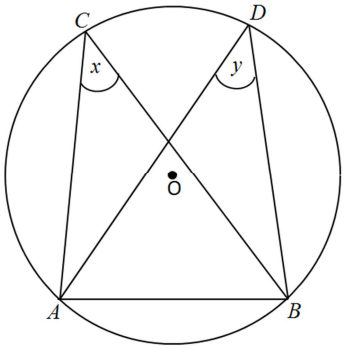

Figure 7. *Angles in the same segment.*

Based on activity two that stated that the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference, with the guidance of the researchers, the students were able to conclude from *Figure 7*, that the two angles ∠ACB and ∠ADB are subtended by the same minor arc AB. Each angle is half the angle ∠AOB at the centre subtended by the same arc. The only way both statements can be true is if angles x and y are equal, so ∠ACB = ∠ADB.

Activity four: Opposite angles in a cyclic quadrilateral is supplementary (sum up to 180°).

Another student led the group to draw a circle and its centre. The students further discussed how to choose four points on the circumference of the circle and connected them with lines to form a cyclic quadrilateral. The groups communicated to themselves to be certain if they were doing the right thing.

Observations:

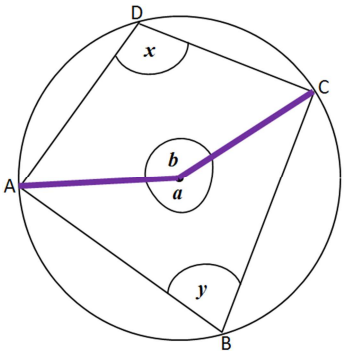

Figure 8. *Cyclic quadrilateral.*

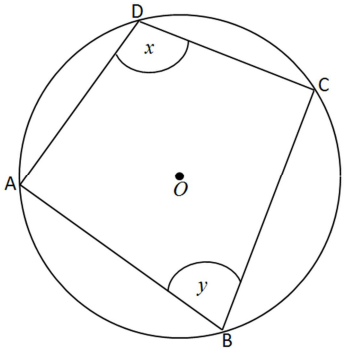

Figure 9. *Opposite angles in a cyclic quadrilateral.*

The students recalled that angles round a point add up to 360° so $2x + 2y = 360°$. With assistance from the researchers, the students discovered that opposite angles in a cyclic quadrilateral is supplementary. Therefore, $x + y = 180°$.

Activity five: An exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.

Students were already familiar with drawing of circles. After they had drawn their circles, students discussed how to locate some points (A, B, C, D) on the circumference of their circles and joined the points to form a cyclic quadrilateral. With instructions from the researchers, each member of the group was told to extend the line BC to a point E, outside the circle. Other members in the groups were asked to measure angles DCE and DAB.

Observations:

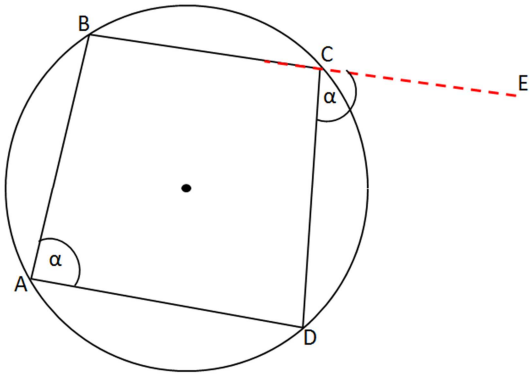

Figure 10. *An exterior angle of a cyclic quadrilateral.*

Students noticed after measuring, that the angles indicated were the same.

Thus ∠DCE = ∠DAB. They also noticed that when directions were changed, they obtained the same result. Team members then became aware that an exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.

Activity six: Equal chords of a circle subtend equal angles at the centre.

Interactively, students drew a circle and drew two chords AB and CD of the same length. Students supported each other in joining the points A, B, C, and D to the centre O. Team members contributed as they measure angles AOB and COD.

Observations:

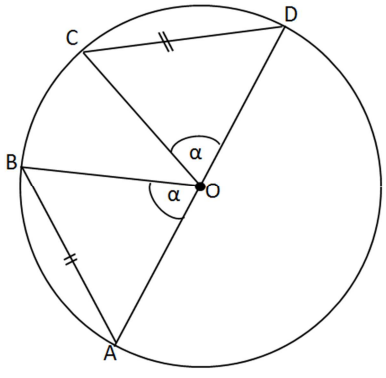

Figure 11. *Equal chords of a circle.*

Students identified that when the circle is rotated so that the arc AB coincides with the arc CD, then the angles $\angle DOC$ and $\angle AOB$ coincide, and hence are equal.

Activity seven

When two chords intersect within a circle, the products of the intercepts are equal.

A member from each group led in discussing the drawing of a circle with two chords AB and CD intersecting at M. The student further explained how to join AC and BD to form two similar triangles. Group members discussed the triangles formed and observed that ΔACM and ΔBDM are similar.

Observations:

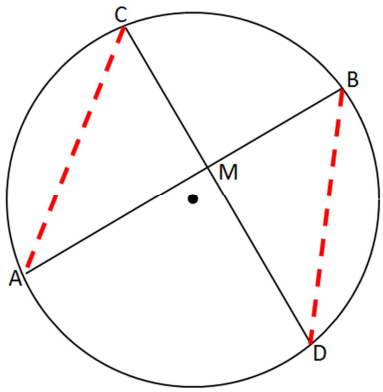

Figure 12. *Intersection of two chords.*

Students shared ideas on triangles AMC and BMD, that $\angle AMC = \angle BMD$ (vertically opposite), $\angle ACD = \angle ABD$ (angles in the same segment), $\angle BAC = \angle CDB$ (angles in the same segment). With assistance from the researchers, students discussed that triangle ACM is similar to triangle BDM, and concluded that $MA \cdot MB = MC \cdot MD$

Activity eight: The products of the intercepts of two secants from an external point are equal.

Group members shared ideas on how to draw a circle with chords AB and CD located in it. They further discussed how to extend chords AB and CD to intersect outside the circle at E. Students then interactively, joined BC and AD to form similar triangles BCE and ADE.

Observations:

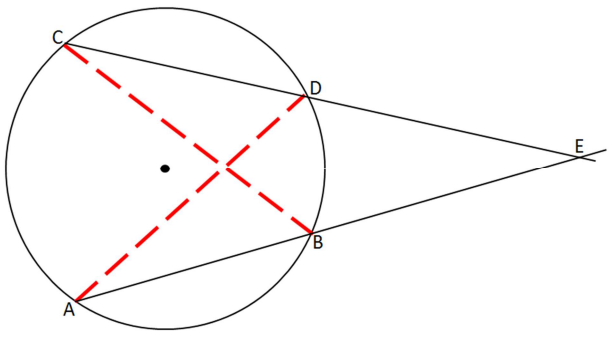

Figure 13. *Intersection of two chords outside a circle.*

Students recalled from the previous activity (activity seven), and contributed that since ΔADE and ΔBCE are similar, it was clear that $EA \cdot EB = EC \cdot ED$

Activity nine: The angle between a tangent and a chord drawn from the point of contact is equal to the angle in the same segment.

Team mates shared thoughts on how to construct a triangle in a semi-circle. Team mates supported each member as they add tangent to the diameter forming the semi-circle. The researchers assisted groups to discuss that tangents always form right angles with diameters and radii. Students noticed that the angles between OC and the tangent are 90°. Interactively, the researchers and the groups identified ∠ BCA + β = 90°.

Observations:

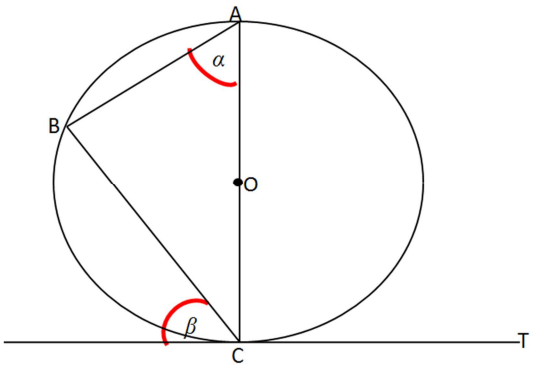

Figure 14. *Angle between a tangent and a chord.*

Students shared that the angle subtended at the circumference by a semicircle is always a right angle, so ∠ ABC = 90°. They also discussed that the interior angles in any triangle add up to 180°. Therefore ∠ABC + ∠BCA + α = 180°, so 90°+ ∠BCA + α = 180°. Students finally explained that ∠BCA + α = 90°, hence α = β.

Activity ten: The two tangents drawn from an external point to a circle are of the same length.

Group members discussed amongst themselves how to draw a circle with suitable radii, OA and OB. Members contributed as they located a point C, outside the circle and drew a tangent AC at A and BC at B, to meet at C. students share idea as they measure AC and BC

Observations:

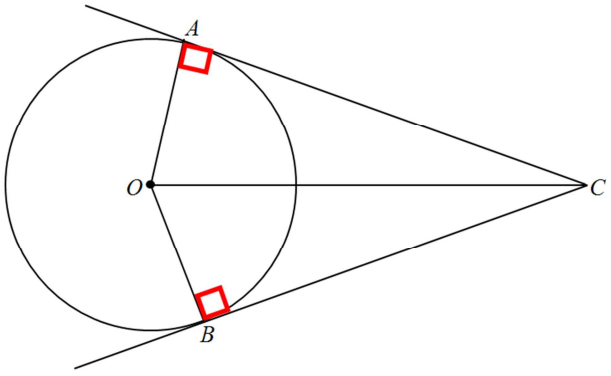

Figure 15. *The two tangents from an external point of a circle.*

Students discussed the following observations:

Triangle *CAO* is congruent (similar) to triangle *CBO* as *OC* is a common side.

∠*CAO* = ∠*CBO* = 90◦, *OA* = *OB* (radii) and therefore *AC* = *BC*.

After the intervention, the researchers wrote all the theorems discussed on twelve different pieces of papers. The researchers then asked members from each group to pick one. Each group was to go over the discussion and make a presentation on the theorems picked, for awarding of marks. Each group did present what they picked, and marks were awarded them according to their level of presentation.

The pre-test and post-test were marked and scored out of 50 as follows;

1. The pre-test was made up of 5 objective tests and 2 essay-type questions.
2. The post-test consisted of 3 essay type questions.

The responses to the items of the questionnaires were analysed using;

Strong Agree (SA), Agree (A), Neutral (N), Disagree (D), Strongly Disagree (SD)

The number for each respondent was summed up to obtain their raw perception scores. The total scores were taken as measures of their perceptions on cooperative learning strategy. The data was displayed using frequency tables. Descriptive statistics including means and standard deviations were used to analyse the pre and post-tests results. Two-tailed test was then used in testing the hypothesis.

4. Findings and Discussions

This section focuses on the results of the analyses of the data and discussions of the findings. The analysis was done based on the research question of the study. The data were organized and presented using both descriptive and inferential approaches of statistics. In the descriptive analysis, a simple procedure of percentage comparisons was used together with pictorial representations in the form of bar graph and frequency tables. The Two-tailed test was used in the inferential statistics to test the null hypothesis: Ho: There is no significant difference between the mean scores of students' performance, when students are taught Circle Theorem using cooperative instructional strategies. $H_{a:}$ Null hypothesis: There is significant difference between the mean scores of students' performance, when students are taught Circle Theorem using cooperative instructional strategies. In all sixty (60) questionnaires were administered to students in Form Two (2) Business class at Brentu Senior High Technical School. Out of this number, 56 questionnaires were returned by the respondents, indicating 93.33% of returns.

Table 1. *Frequency Distribution of Pre-Test Scores.*

Score	Frequency	Percent	Valid Percent	Cumulative Percent
1-10	5	8.3	8.3	8.3
11-20	11	18.3	18.3	26.7
21-30	9	15.0	15.0	41.7
31-40	12	20.0	20.0	61.7
41-50	6	10.0	10.0	71.7
51-60	7	11.7	11.7	83.3

Score	Frequency	Percent	Valid Percent	Cumulative Percent
61-70	5	8.3	8.3	91.7
71-80	5	8.3	8.3	
TOTAL	60	100	100	100.0

From Table 1, of the pre-test, there is a clear indication that students who had more than 50% (pass mark) of the test were 17 (7+5+5), representing 28.33%. Also, students who scored below the pass mark (50%) were 43 (6+12+9+11+5), which also represented 71.67%. This suggests that a whole, 71.67% of the students that participated in the pre-test scored below 50%. The table also shows that the majority of the students (12), scored between 31-40, representing 20% of the total sample.

These results indicated that, students' knowledge and understanding was very limited. The main difficulty of the students which resulted in their poor performance in the pre-test was their inability to apply the concepts, principles and theorems of circles to solve related problems. It was also clear during the analysis of the solutions that students either misrelated the circle theorems to their equation or did not indicate a theorem to signify the mathematical actions taken. Few students therefore were able to obtain correct answers as shown in Table 1 above.

Table 2. *Frequency Distribution of Post-Test Scores.*

Score	Frequency	Percent	Valid Percent	Cumulative Percent
11-20	2	3.3	3.3	3.3
21-30	2	3.3	3.3	6.7
31-40	6	10.3	10.3	16.7
41-50	12	20.0	20.0	36.7
51-60	13	21.7	21.7	38.3
61-70	11	18.3	18.3	76.7
71-80	8	13.3	13.3	95.0
81-90	3	5.0	5.0	95.0
91-100	3	5.0	5.0	
Total	60	100.0	100.0	100.0

The post-test was conducted after an intervention; to address the poor performance of the students in the pre-test was carried out by the researchers. The frequency distribution table of the post-test shows that 38 (13+11+8+3+3) students who participated in the test, representing 63.33% scored more than 50% (pass mark). Also, the majority of the students (13) scored between 51-60, representing 21.67%. After the pre-test had helped the researchers to identify and diagnose the difficulties encountered by students in the circle theorem concept, the post-test helped to determine whether the intervention was successfully carried out. However, upon the analysis of both tests results, it was realised that the use of cooperative learning strategy actually helped the students' understanding of the circle theorem concepts.

Table 3. *Comparison of Pre-Test and Post-Test Results.*

Marks	Pre-Test	%	Post-Test	%
1-10	5	8.3	0	0
11-20	11	18	2	3.3
21-30	9	15	2	3.3
31-40	12	20	6	10
41-50	6	10	12	20
51-60	7	12	13	22
61-70	5	8.3	11	18
71-80	5	8.3	8	13
81-90	0	0	3	5
91-100	0	0	3	5
TOTAL	60	100	60	100

From the above distribution, the analysis of the students' results from both pre-test and post-test gives a clear indication that the intervention really brought about the understanding of the Circle Theorem concept. For instance, in the pre-test 11 students scored between 11% to 20%, representing 18%, whiles in the post only two students scored between 11% to 20%, representing 3.3%. Also, in the pre-test, no student scored above 80%, representing 0%, whereas, in the post-test, 6 students scored more than 80%, representing 10% of the total sample. The Bar Chart below shows the pictorial representation of the comparison in percentage of both the pre-test and post-test.

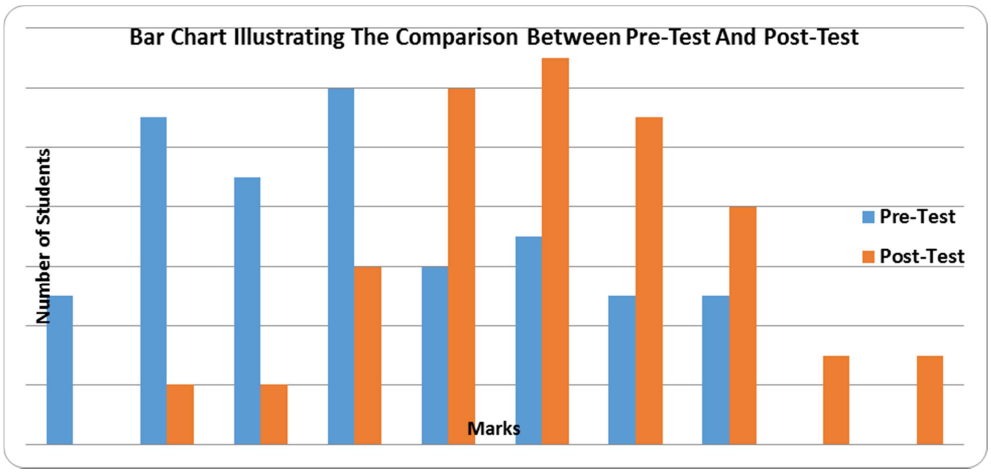

Figure 16. *Bar Chart.*

From the analysis of the scores obtained, the least score in the pre-test was 4% and that of the post-test was 16%. One the other hand, the highest score for the pre-test was 74% and that of the post-test was 94%. When the scores of both pre-

test and post-test were compared and analysed, it was observed that there was difference. In the pre-test, only 28.33% had the pass mark while 63.33% had the pass mark in the post-test (35% difference). This is an indication that students scored higher marks after the introduction of cooperative learning strategy in the intervention stage. Hence, there was difference in the performance of students taught using cooperative learning strategy.

Table 4, below are the results obtained from questionnaire that students were required to express their level of agreement or disagreement with some given statements the use of cooperative learning strategy in the classroom have more of a positive effect on a students' mathematics achievement.

Table 4. *Students' Perceptions on Learning Through Cooperative Strategy.*

Responses	Number of Respondents	%
Strongly Agree	19	34
Agree	26	46
Neutral	4	7.1
Disagree	5	8.9
Strongly Disagree	2	3.6
TOTAL	56	100

It was realised that 34% of the respondents strongly agreed to the items and 46% also agreed. In all, 45 out of the 56 returned questionnaires, representing 80% were in support of the Cooperative learning strategy. However, 5 respondents disagreed and 2 respondents strongly disagreed. These indicated that 7 out of the 56 returned questionnaires, representing 12.5% were not in support of the Cooperative learning strategy. Four (4) respondents, representing 7.1% were not certain on their perceptions on Cooperative learning strategy. An analysis of the perceptions of students about cooperative learning shows that majority of the students perceived cooperative learning to be an effective tool that aided their understanding of the circle theorems. Students became more actively engaged in deducing the circle theorems through cooperative learning strategy during the intervention. The students' activeness reflected in their high scores in the post-test, indicating that, the use of cooperative learning strategy in the classroom have more of a positive effect on a students' mathematics achievement.

Table 4, the interaction effects of students' achievement levels (high, medium and low) when they are taught mathematics using cooperative learning strategy

Table 5. *Students' Responses During Intervention.*

Students' Response	Number of Responses	%
Excellent	4	6.67
Very Good	18	30
Good	29	48.33
Satisfactory	9	15
Poor	0	0
TOTAL	60	100

The table above indicates that 4 students (6.67%) contributed excellently to questions and 18 students (30%) responded very good during the intervention stage. Also, 29 students (48.3%), which is the majority, were good at their responses whereas 9 students (15%) contributed satisfactorily. In contrast to the students' behaviour during whole-class discussions, the targeted students appeared much more engaged when they worked with a partner. The students talked with their partners and shared in handling the manipulatives. They also seemed attentive, making eye contact and nodding in agreement, as other students solved problems or offered explanations, which resulted in effective responses as shown in Table 5. above. This shows that the interaction effects of students' achievement levels would be high when students are taught mathematics using cooperative learning strategy. To test for the hypothesis, the data was analysed using two-tailed test with Statistical Package for Social Sciences (SPSS) version 17, at 0.05 significant level.

Null hypothesis: There is no significant difference between the mean scores of students' performance, when students are taught Circle Theorem using cooperative instructional strategies (i.e. $\mu_1 = \mu_2$). Alternative hypothesis: There is significant difference between the mean scores of students' performance, when students are taught Circle Theorem using cooperative instructional strategies (i.e. $\mu_1 \neq \mu_2$).

Table 6. *Descriptive Statistics of Students' Performance in Pre-test and Post-test.*

	N	Minimum	Maximum	Mean	Std. Deviation
Pre-Test Scores	60	4	74	35.20	20.535
Post-Post Scores	60	16	94	57.27	17.829
Valid N (listwise)	60				

The output in Table 6. indicates that there were 60 observations (N) in both the pre-test and post-test. The mean of the pre-test was 35.20 with a standard deviation of 20.54 and the mean for the post-test was 57.27b with a standard deviation of 17.83. A Two-tailed test was run to determine whether there was significant difference in the mean scores of both the pre-test and the post-test. The results are presented in Table 7. Below.

Table 7. *Two-tailed Test For Students' Performance in both Pre-test and Post-test.*

	Test Value = 50					
	T	df	Sig. (2-taileded)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
Pre-Test	-5.583	59	.000	-14.800	-20.10	-9.50
Post-Test	3.157	59	.003	7.267	2.66	11.87

The test value (50) was the pass mark the pre-test and post-test were compared to. The first three columns of Table 7 provide information on whether the means of both pre-test and post-test were statistically significantly different. Moving from right to left, the observed t–value for the pre-test was t = -5.583 with a degree of freedom (df), 59 (N – 1) and a statistical significance (p – value) i.e. [Sig. (2-tailed)] of 0.000. Also, the observed t–value for the post-test was t = 3.157 with a (df) of 59 and a p–value of 0.003. In both cases p–value was less than 0.05. This result is statistically significant since the p–value is less than the significant level (0.05). Statistically, if $p < 0.05$, then the null hypothesis can be rejected. Therefore, there is sufficient evidence to conclude that there is significant difference between the mean scores of students' performance, when students are taught Circle Theorem using cooperative instructional strategies.

The main purpose of this study was to identify the impact of cooperative learning strategy on mathematics achievements of students in Brentu Senior High Technical School, Enchi. The results, therefore, support the notion that cooperative and collaborative learning can facilitate the acquisition of knowledge, recognized as valuable components of classroom learning [15]. One of the reasons for the outcome may be due to the interactions between the students. The study demonstrated the effectiveness of cooperative learning in the teaching and learning of mathematics at the senior high school level of education. This study is most significant in that it demonstrated;

1. The extent to which the use of cooperative learning strategy could help students' understanding of circle theorem.
2. The differences in the performance of students taught Circle Theorem using cooperative and individualistic instructional strategies.
3. The extent to which the use of cooperative learning strategy in the classroom would have more of a positive effect on a students' mathematics achievement.
4. The interaction effects of students' achievement levels (high, medium and low) when they are taught mathematics using cooperative learning strategy.

One major finding of this study was that students taught using the cooperative learning approach scored higher marks in the post-test than in the pre-test. This may have been achieved by the high level of students' participation in learning activities. All the students in the cooperative group performed specific roles in solving problems which are presented in the classroom to the benefit of all members of the group. When learners are confronted with problems which they must solve, they are forced to reason and think critically in order to solve the problems. It is believed that when properly and carefully used cooperative learning activities engage the students in the learning process and seek to improve the critical thinking, reasoning and problem-solving skill of learners [31]. Specifically, these research findings are hinged on certain principles which guided the studies. A study on the principle hinged by [5] that cooperative learning enhances active engagement of students and critical thinking. Most importantly, constructive activity and collaborated learning are enhanced in cooperative learning classrooms [31]. This study also revealed that students in the cooperative learning classroom were found to exhibit better attitude towards the learning of mathematics (circle theorems), as measured by their attitude scores, using an attitude scale. This seems to agree with the general notion that individuals can change their attitude and disposition through interaction with others in one way or the other [4], for example, noted that cooperative learning is important in helping learners acquire from the curriculum the basic cooperative attitudes and values they need in the classroom and outside the classroom. The attitudes exhibited by students in the cooperative learning class may also be explained, at least in part, by the fact that interaction among students in cooperative learning groups is intense and prolonged.

The findings further revealed that in classes where cooperative learning approach is used for teaching, students gradually take responsibility for each other's learning. The better attitude exhibited by students in the cooperative learning classroom may have been achieved because feedback, reinforcement, and support come from students' peers in the group. Again, students in cooperative learning performing better in test of attitude towards studies may perhaps be because of imbibing of role expectations and responsibility, which are two very important features of cooperative learning. The findings of the study posit that the success of a cooperative learning activity depends on your communicational role, expectations, responsibilities and modelling them where necessary [4].

The main difficulty of the students which resulted into the low achievement in the pre-test was their inability to apply the concepts, principles and theorems of circle to solve related problems. The difficulties encountered by students were not different from previous study which proposed that students' difficulties were as a result of memorizing properties rather than exploring to discover properties and misuse of geometrical terminologies often leads to misconceptions of geometrical knowledge [26]. The students' difficulties in the pre-test leading to their low achievements in circle theorems is a clear manifestation of the students' difficulties in answering circle theorem questions as indicated in the Chief Examiner's Report [30]. However, working together affords the opportunity to share ideas and strategies for learning new things. Another possibility could be that being in a group allows you to remember the information better because you can see the implementation of the practice rather than learning the material independently. The positive effects of cooperative learning are evident and there is little question that cooperative learning can be a useful tool and should be utilized in the learning environment.

5. Conclusions and Recommendations

It is evidence from the study that cooperative learning is a feasible and practical teaching method that puts communicative approach into action. Such a student-centred teaching method helps improve the students' communicative competence. Cooperative learning creates natural, interactive contexts in which students have authentic reasons for listening to one another, asking questions, clarifying issues, and re-stating points of view. Such frequent interaction among the learners, in turn, increases the amount of student talk and student participation in the classroom. The study further concluded that, cooperative learning enables teachers to get through to student with different learning strengths. Group learning is more student-centred and engages students in active learning. As a result, it prepares learners to enter the modern workforce, where people are expected to be able to solve problems effectively and work collaboratively with others.

It is recommended that, mathematics teachers should employ cooperative learning strategies to improve students' performance to bridge the gap among high, medium and low achievers. It is further also recommended that more emphasis should be given on students' understanding of a particular concept, guiding students in active learning, providing opportunities for discussion and elaboration and encouraging them to work with peers and teachers. It is further recommended that teachers at the Senior High School level should be trained on implementation of cooperative learning strategies so as to improve their performance in mathematics.

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