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Carnot Heat Engine with a Variable Generalized Chaplygin Gas

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Abstract

Observational evidences suggest that the Universe is accelerating and the cosmological Chaplygin gas model is one of the most reasonable explanations of this phenomena. This model allows to simulate the dark energy in the cosmic fluid and has a great application in the study of the fundamental theories of physics and cosmology. Several independent observations indicate that the greater part of the total energy density of the universe is in the form of a dark energy and the rest in the form of non-baryonic cold dark matter particles, but which have never been detected. In this paper, following usual procedure has been extended the work of Panigrahi and Chatterjee (2016) for a variable generalized Chaplygin gas and has been studied the thermodynamical behavior for a Carnot engine using the thermal equations of state for the pressure and internal energy as function of temperature and volume for this type of gas. It has been derive an expression for the thermal efficiency of Carnot heat engine that depends on the limits of maximum and minimal temperature imposed on the cycle and of an exponent associated with the equation of state of variable generalized Chaplygin gas. Depending on the value of the exponent is recovered the expression for the efficiency of a Carnot cycle in an ideal gas as a particular case of this work.

Keywords

Variable Generalized Chaplygin Gas, Dark Energy, Thermal Equation of State, Carnot Heat Engine, Thermal Efficiency

1. Introduction

The discovery of accelerating expansion of the universe [1–3] has allowed generated important changes in the fundamental theories in physics and cosmology and a Chaplygin type of gas cosmology is one of the most reasonable explanations for this phenomena. Astronomical observations [4, 5] show that the kind of matter of which stars and galaxies are made forms less than 5% of the universe's total mass and the independent observations indicate that the greater part of the total energy density of the universe is in the form of a dark energy, and the rest in the form of non-baryonic cold dark matter particles, but which have never been detected [6].

The thermodynamical behaviour of the Chaplygin gas model was studied by Myung [7] and Panigrahi [8]. Myung [7] found a new general equation of state that describes the Chaplygin gas completely. Panigrahi [8] obtains that the third law of thermodynamics is satisfied in this model and that the volume increases when temperature falls during adiabatic expansions, which also is observed in an gas ideal [9]. Malaver [10] found that the thermodynamic efficiency of Carnot cycle for CG model only depend on the limits of maximum and minimal temperature as in case of the ideal gas and the photons gas. More recently, Panigrahi and Chatterjee [6] studied the viability of the variable generalized Chaplygin gas (VGCG) whose equation of state is the following $P=-\frac{B}{\rho^{\alpha}}$ where P is the pressure of the fluid, ρ is the energy density, α is a parameter and $B=B_O V^{-\frac{n}{3}}$ where n is an arbitrary constant and V is the volume of the fluid and derived thermodynamic expressions as functions of temperature and volume.

An ideal gas is a gas composed of a group of randomly moving, non-interacting point particles. The ideal gas approximation is useful because it obeys the gases laws and represent the vapor phases of fluids at high temperatures for which the heat engines is constructed [11]. Any device for converting heat into work in a cyclic process can be called a heat engine or thermal machine and must operate in the presence of two different temperatures [12]. In a steam engine, for example, the high temperature is the temperature of the steam and the low temperature is the condensed cold. A heat engine that can work with an ideal gas as working substance is the Carnot cycle. For the ideal gas, Carnot cycle will be composed by two isothermal curves and adiabatic which will come given by the conditions $PV = const$ and $PV^{\gamma} = const$, respectively [12, 13] where γ is an adiabatic exponent.

One of the great virtues of the Carnot cycle is its potential applicability to any working substance [13]. In agreement with Leff [9] and Lee [11] the Carnot cycle for a photon gas provides a very useful tool to illustrate the thermodynamics laws and it is possible to use for introducing the concepts of creation and annihilation of photons in an introductory course of physics. Bender et al. [14], showed that the efficiency of a quantum Carnot cycle is the same as that of a classical Carnot cycle, with the identification of the expectation value of the Hamiltonian as the temperature of the system. Unlike the ideal gas, the pressure for a photon gas is a function only of the temperature and the internal energy function is dependent of volume [9].

In this paper, an expression of the efficiency of Carnot cycle for the VGCG model is deduced with a thermal equation of state given for Panigrahi and Chatterjee [6] that depends of the temperature and volume. It has been found that the efficiency of Carnot cycle for the VGCG model will depend on the limits of maximum and minimal temperature imposed on the cycle and the parameter α. The article is organized as follows: in Section 2, the physical properties of Carnot heat engine are studied; in Section 3, is shown the deduction for the thermal efficiency of Carnot cycle for the ideal gas; in Section 3, is obtained an expression for the efficiency of Carnot engine for the VGCG model; in Section 4, presents the conclusions of this study.

2. Carnot Heat Engine

In the process of operation of a heat engine between two different temperatures, some heat is always transferred on the outside [12]. If an amount of heat Q_H is absorbed at the high temperature the work is done on the surroundings, and if a quantity of heat Q_L is lost at the lower temperature, then of the first law of thermodynamics for the entire cyclic process $Q_H + Q_L + W_{net} = \Delta U = 0$ where ΔU is the variation of the internal energy in the process and W_{net} is the work in the cycle [12, 13]. The efficiency of a heat engine is commonly defined as the ratio of the work obtained from the system to the heat taken from the hot reservoir

$$\eta = \frac{-W_{net}}{Q_H} = \frac{Q_H + Q_L}{Q_H} = 1 + \frac{Q_L}{Q_H} \tag{1}$$

A particularly simple heat engine cycle to handle mathematically is the Carnot cycle [12, 15]. In the Figure 1, two temperatures are included, T_H and T_L. The first step in a Carnot cycle is a reversible isothermal expansion a T_H or from point A to point B. This expansion could be achieved by expanding the gas in contact with a large heat reservoir at T_H. A certain amount of work will be done on the surroundings which implies an absorption of heat.

The second step is an reversible adiabatic expansion from the state at point B at point C. Under these conditions $Q_{BC} = 0$ and $\Delta U_{BC} = W_{BC}$ and the internal energy change is the same as the work done on the working substance. Since work is done on the surroundings, W_{BC} is negative and the internal energy must fall.

The third step, the reversible isothermal compression, is continued just to the point C where a final adiabatic compression will bring the gas back to its starting conditions at point A on the PV plot of the Figure 1. Work W_{CD} is done on the gas, and an amount of heat Q_L is lost from the gas which compensates for this work exactly in an ideal gas and approximately in a real gas.

In agreement with Dickerson [12] the final adiabatic compression to the starting point occurs with work W_{DA} done on the gas and an increase in the internal energy. For the entire cycle, $W_{net} = W_{AB} + W_{BC} + W_{CD} + W_{DA}$ and $Q_{neto} = Q_H + Q_L$. The total sum of heat and works is zero since the initial and final states are identical

$$Q_H + Q_L + W_{AB} + W_{BC} + W_{CD} + W_{DA} = Q_{net} + W_{net} = \Delta U = 0 \tag{2}$$

The efficiency of the entire cycle in converting heat to work is

$$\eta = \frac{-W_{net}}{Q_H} = \frac{Q}{Q_H} = \frac{Q_H + Q_L}{Q_H} = 1 + \frac{Q_L}{Q_H} \tag{3}$$

Since Q_H and Q_L have opposite signs, the efficiency is less than 1 and is the greatest possible efficiency.

3. Carnot Cycle in an Ideal Gas

In this work, we have used the convention of Wark and Richards [15] that defines the work during a reversible process as

$$W = -\int PdV \tag{4}$$

Following Dickerson [12] and Nash [13], in Figure 1 we show the Carnot cycle for an ideal gas. In the first step of A to B, that is the isothermal expansion, there is no change in the internal energy ΔU in an ideal gas. This implies that

$$-W_{AB} = Q_{AB} = RT_H \ln \frac{V_B}{V_A} \tag{5}$$

Q_{AB} is the absorbed heat in the first step, T_H is the high temperature and R is the universal gas constant.

The second step of B to C is an adiabatic expansion. In this expansion $Q_{BC}=0$ and the change in internal energy is equal to the work done

$$\Delta U_{BC}=W_{BC}=C_V\left(T_L-T_H\right) \tag{6}$$

where C_V is the thermal capacity at constant volume and T_L is the low temperature.

In the isothermal compression of C to D, the internal energy change is again zero and we obtain

$$-W_{CD}=Q_{CD}=RT_L\ln\frac{V_D}{V_C} \tag{7}$$

In the final adiabatic compression of D to A $Q_{DA}=0$ and

$$\Delta U_{DA}=W_{DA}=C_V\left(T_H-T_L\right) \tag{8}$$

In a Carnot cycle for an ideal gas the net work done in the two adiabatic processes is zero and the adiabatic steps are related by the equation

$$\frac{V_B}{V_C}=\frac{V_A}{V_D}=\left(\frac{T_L}{T_H}\right)^{C_V/R} \tag{9}$$

The network of the four steps is $W_{net}=W_{AB}+W_{CD}$. Substituting (9) into eqs. (5) and (7), we obtain

$$W_{net}=-R\left(T_H-T_L\right)\ln\frac{V_A}{V_B} \tag{10}$$

The heat absorbed at the high temperature is Q_{AB} and the thermal efficiency of the entire cycle is

$$\eta=\frac{-W_{net}}{Q_{AB}}=1-\frac{T_L}{T_H} \tag{11}$$

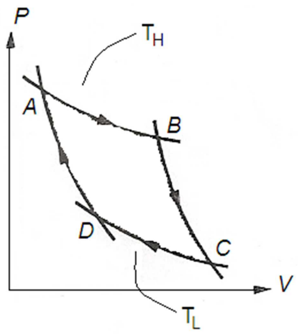

Figure 1. *Carnot cycle for an ideal gas.*

4. Thermal Efficiency in a Variable Generalized Chaplygin Gas

According Panigrahi and Chatterjee [6], the equations of state for the internal energy and pressure as a function of V and T for the VGCG model can be written as

$$U=V\left[\frac{\dfrac{B_0\left(1+\alpha\right)V^{-\frac{n}{3}}}{N}}{1-\left(\dfrac{T}{\tau}\right)^{\frac{1+\alpha}{\alpha}}}\right]^{\frac{1}{1+\alpha}} \tag{12}$$

$$P=-\left(B_0V^{-\frac{n}{3}}\right)^{\frac{1}{1+\alpha}}\left(\frac{N}{1+\alpha}\right)^{\frac{\alpha}{1+\alpha}}\left\{1-\left(\frac{T}{\tau}\right)^{\frac{1+\alpha}{\alpha}}\right\}^{\frac{\alpha}{1+\alpha}} \tag{13}$$

where B_0 is a positive universal constant, n is a constant, $N=\dfrac{3\left(1+\alpha\right)-n}{3}$ and τ is a universal constant with dimension of temperature.

Considering the Carnot cycle for a VGCG model, in the first step, the reversible isothermal expansion and substituting (13) in (4) and integrating, the work done is

$$W_I=\left[\frac{B_0\left(1+\alpha\right)}{N}\right]^{\frac{1}{1+\alpha}}\left[1-\left(\frac{T_H}{\tau}\right)^{\frac{1+\alpha}{\alpha}}\right]^{\frac{\alpha}{1+\alpha}}\left(V_B^{\frac{N}{1+\alpha}}-V_A^{\frac{N}{1+\alpha}}\right) \tag{14}$$

The expression for the differential of internal energy dU is given by

$$dU=\left(\frac{\partial U}{\partial T}\right)_V dT+\left(\frac{\partial U}{\partial V}\right)_T dV \tag{15}$$

where $C_V=\left(\dfrac{\partial U}{\partial T}\right)_V$

For an isothermal process $dT=0$ and (15) it reduce to

$$dU=\left(\frac{\partial U}{\partial V}\right)_T dV \tag{16}$$

and for VGCG gas, $\left(\dfrac{\partial U}{\partial V}\right)_T$ takes the form

$$\left(\frac{\partial U}{\partial V}\right)_T=\left[\frac{3(1+\alpha)-n}{3(1+\alpha)}\right]V^{\frac{3(1+\alpha)-n}{3(1+\alpha)}-1}\frac{\left[\frac{B_0(1+\alpha)}{N}\right]^{\frac{1}{1+\alpha}}}{\left[1-\left(\frac{T_H}{\tau}\right)^{\frac{1+\alpha}{\alpha}}\right]^{\frac{1}{1+\alpha}}} \tag{17}$$

Substituting (17) in (16) and integrating it is obtained for the change of internal energy

$$\Delta U_I=\left[\frac{B_0(1+\alpha)}{N}\right]^{\frac{1}{1+\alpha}}\left[1-\left(\frac{T_H}{\tau}\right)^{\frac{1+\alpha}{\alpha}}\right]^{-\frac{1}{1+\alpha}}\left(V_B^{\frac{N}{1+\alpha}}-V_A^{\frac{N}{1+\alpha}}\right) \tag{18}$$

With the eq. (14), (18) and the first law of thermodynamics [12] the absorbed heat in the first step is given by

$$Q_I=\left[\frac{(1+\alpha)B_0}{N}\right]^{\frac{1}{1+\alpha}}\left(\frac{T_H}{\tau}\right)^{\frac{\alpha+1}{\alpha}}\frac{\left(V_B^{\frac{N}{1+\alpha}}-V_A^{\frac{N}{1+\alpha}}\right)}{\left[1-\left(\frac{T_H}{\tau}\right)^{\frac{\alpha+1}{\alpha}}\right]^{\frac{1}{\alpha+1}}} \tag{19}$$

In the third path, the reversible isothermal compression, the work done is

$$W_{III}=\left[\frac{B_0(1+\alpha)}{N}\right]^{\frac{1}{1+\alpha}}\left[1-\left(\frac{T_L}{\tau}\right)^{\frac{1+\alpha}{\alpha}}\right]^{\frac{\alpha}{1+\alpha}}\left(V_D^{\frac{N}{1+\alpha}}-V_C^{\frac{N}{1+\alpha}}\right) \tag{20}$$

and for the transferred heat Q_{III} is obtained

$$Q_{III}=\left[\frac{(1+\alpha)B_0}{N}\right]^{\frac{1}{1+\alpha}}\left(\frac{T_L}{\tau}\right)^{\frac{\alpha+1}{\alpha}}\frac{\left(V_D^{\frac{N}{1+\alpha}}-V_C^{\frac{N}{1+\alpha}}\right)}{\left[1-\left(\frac{T_L}{\tau}\right)^{\frac{\alpha+1}{\alpha}}\right]^{\frac{1}{\alpha+1}}} \tag{21}$$

The network of the four steps is

$$W_{neto}=W_I+W_{II}+W_{III}+W_{IV} \tag{22}$$

and the net heat is

$$Q_{neto}=Q_I+Q_{III} \tag{23}$$

For a cyclical process [12, 13], $\Delta U=0$ and the net work can be written as

$$W_{neto}=-Q_{neto}=\left[\frac{B_0(1+\alpha)}{N}\right]^{\frac{1}{1+\alpha}}\left[\frac{\left(\frac{T_H}{\tau}\right)^{\frac{1+\alpha}{\alpha}}}{\left[1-\left(\frac{T_H}{\tau}\right)^{\frac{1+\alpha}{\alpha}}\right]}\left(V_A^{\frac{N}{1+\alpha}}-V_B^{\frac{N}{1+\alpha}}\right)+\frac{\left(\frac{T_L}{\tau}\right)^{\frac{1+\alpha}{\alpha}}}{\left[1-\left(\frac{T_L}{\tau}\right)^{\frac{1+\alpha}{\alpha}}\right]}\left(V_C^{\frac{N}{1+\alpha}}-V_D^{\frac{N}{1+\alpha}}\right)\right] \tag{24}$$

The heat absorbed at T_H is given by (19) and the efficiency is

$$\eta = \frac{-W_{neto}}{Q_I} = 1 - \left(\frac{T_L}{T_H}\right)^{\frac{1+\alpha}{\alpha}} \frac{\left[1-\left(\frac{T_H}{\tau}\right)^{\frac{1+\alpha}{\alpha}}\right]^{\frac{1}{1+\alpha}}}{\left[1-\left(\frac{T_L}{\tau}\right)^{\frac{1+\alpha}{\alpha}}\right]^{\frac{1}{1+\alpha}}} \frac{\left(V_D^{\frac{N}{1+\alpha}} - V_C^{\frac{N}{1+\alpha}}\right)}{\left(V_A^{\frac{N}{1+\alpha}} - V_B^{\frac{N}{1+\alpha}}\right)} \tag{25}$$

eq. (25) can be written as

$$\eta = \frac{-W_{neto}}{Q_I} = 1 - \left(\frac{T_L}{T_H}\right)^{\frac{1+\alpha}{\alpha}} \frac{\left[1-\left(\frac{T_H}{\tau}\right)^{\frac{1+\alpha}{\alpha}}\right]^{\frac{2}{1+\alpha}}\left[1-\left(\frac{T_L}{\tau}\right)^{\frac{1+\alpha}{\alpha}}\right]^{\frac{1}{1+\alpha}}}{\left[1-\left(\frac{T_H}{\tau}\right)^{\frac{1+\alpha}{\alpha}}\right]^{\frac{1}{1+\alpha}}\left[1-\left(\frac{T_L}{\tau}\right)^{\frac{1+\alpha}{\alpha}}\right]^{\frac{2}{1+\alpha}}} \frac{V_C^{\frac{N}{1+\alpha}}\left(\frac{V_D^{\frac{N}{1+\alpha}}}{V_C^{\frac{N}{1+\alpha}}} - 1\right)}{V_B^{\frac{N}{1+\alpha}}\left(\frac{V_A^{\frac{N}{1+\alpha}}}{V_B^{\frac{N}{1+\alpha}}} - 1\right)} \tag{26}$$

For a reversible adiabatic process in the VGCG model [16]

$$\frac{V}{\left[1-\left(\frac{\tau}{T}\right)^{\frac{1+\alpha}{\alpha}}\right]^{1/N}} = const \tag{27}$$

Then of the eq. (27) it is deduced

$$\left(\frac{V_C}{V_B}\right)^{\frac{N}{1+\alpha}} = \left(\frac{T_H}{T_L}\right)^{\alpha} \frac{\left[\left(T_L\right)^{\frac{1+\alpha}{\alpha}} - \left(\tau\right)^{\frac{1+\alpha}{\alpha}}\right]^{\frac{1}{1+\alpha}}}{\left[\left(T_H\right)^{\frac{1+\alpha}{\alpha}} - \left(\tau\right)^{\frac{1+\alpha}{\alpha}}\right]^{\frac{1}{1+\alpha}}} \tag{28}$$

Substituting eq. (28) in eq. (26), the efficiency can express as

$$\eta = \frac{-W_{neto}}{Q_I} = 1 - \left(\frac{T_L}{T_H}\right)^{\frac{1+\alpha}{\alpha}} \frac{\left[1-\left(\frac{T_H}{\tau}\right)^{\frac{1+\alpha}{\alpha}}\right]^{\frac{2}{1+\alpha}}\left[1-\left(\frac{T_L}{\tau}\right)^{\frac{1+\alpha}{\alpha}}\right]^{\frac{1}{1+\alpha}}}{\left[1-\left(\frac{T_H}{\tau}\right)^{\frac{1+\alpha}{\alpha}}\right]^{\frac{1}{1+\alpha}}\left[1-\left(\frac{T_L}{\tau}\right)^{\frac{1+\alpha}{\alpha}}\right]^{\frac{2}{1+\alpha}}} \left(\frac{T_H}{T_L}\right)^{\alpha} \frac{\left[\left(T_L\right)^{\frac{1+\alpha}{\alpha}} - \left(\tau\right)^{\frac{1+\alpha}{\alpha}}\right]^{\frac{1}{1+\alpha}}}{\left[\left(T_H\right)^{\frac{1+\alpha}{\alpha}} - \left(\tau\right)^{\frac{1+\alpha}{\alpha}}\right]^{\frac{1}{1+\alpha}}} \frac{\left(\frac{V_D^{\frac{N}{1+\alpha}}}{V_C^{\frac{N}{1+\alpha}}} - 1\right)}{\left(\frac{V_A^{\frac{N}{1+\alpha}}}{V_B^{\frac{N}{1+\alpha}}} - 1\right)} \tag{29}$$

The eq. (27) implies that for the Carnot cycle in a VGCG model

$$\frac{V_D}{V_C} = \frac{V_A}{V_B} \tag{30}$$

With eq. (30), eq. (29) can be written as

$$\eta=\frac{-W_{neto}}{Q_I}=1-\left(\frac{T_L}{T_H}\right)^{\frac{1+\alpha}{\alpha}}\frac{\left[1-\left(\frac{T_H}{\tau}\right)^{\frac{1+\alpha}{\alpha}}\right]^{\frac{2}{1+\alpha}}\left[1-\left(\frac{T_L}{\tau}\right)^{\frac{1+\alpha}{\alpha}}\right]^{\frac{1}{1+\alpha}}}{\left[1-\left(\frac{T_H}{\tau}\right)^{\frac{1+\alpha}{\alpha}}\right]^{\frac{1}{1+\alpha}}\left[1-\left(\frac{T_L}{\tau}\right)^{\frac{1+\alpha}{\alpha}}\right]^{\frac{2}{1+\alpha}}}\left(\frac{T_H}{T_L}\right)^{\alpha}\frac{\left[\left(T_L\right)^{\frac{1+\alpha}{\alpha}}-\left(\tau\right)^{\frac{1+\alpha}{\alpha}}\right]^{\frac{1}{1+\alpha}}}{\left[\left(T_H\right)^{\frac{1+\alpha}{\alpha}}-\left(\tau\right)^{\frac{1+\alpha}{\alpha}}\right]^{\frac{1}{1+\alpha}}} \tag{31}$$

Rearranging (31), it is deduced for the thermal efficiency in a Carnot cycle for the VCGC model

$$\eta=1-\left(\frac{T_L}{T_H}\right)^{\frac{1+\alpha}{\alpha}}\left(\frac{T_H}{T_L}\right)^{\alpha} \tag{32}$$

With $\alpha=1$ is recovered the efficiency for the Carnot cycle for an ideal gas as a particular case of this work

$$\eta_{ideal}=1-\frac{T_L}{T_H} \tag{33}$$

5. Conclusions

In this work has been deduced an expression for the efficiency of a Carnot heat engine with a variable generalized Chaplygin gas, which is a function of the maximum and minimal temperature of the thermodynamic cycle and the parameter α. The study of Chaplygin gas can enrich the courses of thermodynamics, which contributes to a better compression of the thermal phenomena.

The thermodynamic equations that describe the behavior of the Chaplygin gas are tractable mathematically and offer a wide comprehension of the accelerated universe expansion and of the basic ideas of the modern cosmology.

It is showed that the efficiency of Carnot cycle in a variable generalized Chaplygin gas is the same as that of a classical Carnot cycle when $\alpha=1$ as in the ideal gas and the photon gas. For a Carnot heat engine in the VGCG model to be 100% efficient, temperature of hot reservoir T_H must be infinite or zero for the cold reservoir T_L and the second law says that a process cannot be 100% efficient in converting heat into work.

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