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Construction of Balanced Incomplete Block Design: An Application of Galois Field

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Abstract

This paper focuses on the application of Galois field to construct the balanced incomplete block design. In *GF*(7), minimum function has been calculated, hence generate the element of *GF*(7) and construct mutual orthogonal Latin square (MOLS). Using mutual orthogonal Latin square, balanced incomplete block design has been made.

Keywords

BIBD, Galois Field (GF), MOLS

1. Introduction

In Incomplete block designs, as their name implies, the block size is less than the number of treatments to be tested. These designs were introduced by Yates in order to eliminate heterogeneity to a greater extent than is possible with randomized blocks and Latin squares when the number of treatments is large. If the number of treatments to be compared is large, then we need large number of blocks to accommodate all the treatments. This requires more experimental material and so the cost of experimentation becomes high which may be in terms of money, labor, time etc. The completely randomized design and randomized block design may not be suitable in such situations because they will require large number of experimental units to accommodate all the treatments. In such situations when sufficient numbers of homogeneous experimental units are not available to accommodate all the treatments in a block, then incomplete block designs can be used. In incomplete block designs, each block receives only some of the selected treatments and not all the treatments. Sometimes it is possible that the available blocks can accommodate only a limited number of treatments due to several reasons. The precision of the estimate of a treatment effect depends on the number of replications of the treatment. Similar is the case for the precision of estimate of the difference between two treatment effects. If a pair of treatment occurs together more number of times in the design, the difference between these two treatment effects can be estimated with more precision. To ensure equal or nearly equal precision of comparisons of different pairs of treatment effects, the treatments are so allocated to the experimental units in different blocks of equal sizes such that each treatment occurs at most once in a block and it has an equal number of replications and each pair of treatments has the same or nearly the same number of replications. When the number of replications of all pairs of treatments in a design is the same, then we have an important class of designs called Balanced Incomplete Block (BIB) designs. It was first devised by Yates in 1936 for agricultural experiments. These design have evidently some constructional problems because the allotments of k of the v treatments in different blocks, so that each pair of treatments is replicated a constant number of times is not straight - forward. The constructional problems were solved by the joint efforts of Fisher, Yates and Bose in 1939 among others. While Fisher and Yates being in touch with the experimental scientists were restrained in their efforts to obtain new designs and new analytical techniques by the requirements of the experimenters, Bose constructed more on the methods of construction of the balanced and their

Incomplete Block Designs and was not necessarily constructed by the consideration of practical utility.

Das and Giri [7] discussed the basic concept of design of experiments. Bose [4], Bose *et al.* [5] and Mann [10] discussed construction of mutually orthogonal Latin square. Das and Giri [7] were briefly explained for Latin Square Design, Graeco Latin Squares, introduction to Mutually Orthogonal Latin Square (MOLS), Construction of MOLS (4 x 4) and construction of Greaco Latin Square Design of order only (5 x 5). Discussed about construction and incomplete block design, Bose [3], Connor [6], Fisher [8], Kshirsagar [9], Mann [10], Menon [11] are noteworthy. Sharma and Kumar [14] developed balanced incomplete block design using Hadamard rhotrices. Bayrak and Bulut [2] constructed orthogonal balanced incomplete block design. Arunachalam *et al.* [1] constructed of efficiency-balanced design. Pachamuthu [13] showed construction of mutually orthogonal Latin square and check parameter relationship of balanced incomplete block design.

This paper has been shown the construction of balanced incomplete block design using mutual orthogonal Latin square. To construct mutual orthogonal Latin square Galois field theory has been used. Construction of Galois field $GF\left(p^m\right)$, finding minimum function of Galois field $GF(7)$ and hence construct mutual orthogonal Latin square.

2. Galois Field

Galois Field, named after Evariste Galois, also known as finite field, refers to a field in which there exist finitely many elements. It is particularly useful in translating computer data as they are represented in binary forms. That is, computer data consist of combination of two numbers, 0 and 1, which are the components in Galois field whose number of elements is two. Representing data as a vector in a Galois Field allows mathematical operations to scramble data easily and effectively.

The finite field format by the p^m classes of residues is called a Galois field of order p^m and is denoted by $GF\left(p^m\right)$, where, p is prime number and m is positive integer. The function $p(x)$ is said to be a minimum function for generating the elements of $GF\left(p^m\right)$. The non-zero elements may be represented either as polynomials degree at most $(m-1)$ as we know the power of primitive root x such that $x^{p^m-1}+(p-1)=0$. To obtain the minimum function we divide $x^{p^m-1}+(p-1)$ by the least common multiple of all factors lies x^d+1, where d is a divisor of p^m-1. Then we get the cyclotomic equation. That is, the equation that has for its roots, all primitive roots of the equation $x^{p^m-1}+(p-1)=0$. The order of the equation will be $\phi\left(p^m-1\right)$, where $\phi(k)$ denotes the number of positive integers less than k and relatively prime to it. In this equation, by replacing each coefficient by its least non-zero residue to modulus p. We get the cyclotomic polynomial of order $\phi\left(p^m-1\right)$. Let $p(x)$ be an irreducible factor of this polynomial, then $p(x)$ is a minimum function which is in general not unique.

The elements of Galois field $GF\left(p^m\right)$ is defined as

$$GF\left(p^m\right)=(0,1,2,\ldots,p-1)\cup(p,p+1,p+2,\ldots,p+p-1)\cup\left(p^2,p^2+1,p^2+2,\ldots,p^2+p-1\right)\cup$$
$$\left(p^{m-1},p^{m-1}+1,p^{m-1}+2,\ldots,p^{m-1}+p-1\right)$$

where $p\in \mathrm{P}$ and $m\in \mathrm{Z}^+$. The order of the field is given by p^m while p is called the characteristics of the field.

Example

$$GF(7)=(0,1,2,3,4,5,6)$$

which consists of 7 elements where each of them is a polynomial of degree 0 (a constant).

2.1. Construction of Galois Field *GF*(p^m)

Construction of Galois field of p^m elements from the p^{th} order field $GF(p)$. p^{th} elements of $GF(p)$ are $0,1,\ldots,(p-1)$ and a new symbol α. Then define a multiplication "•" to introduce a sequence of power of α as follows

$$0\bullet 0=0,$$
$$0\bullet 1=1\bullet 0=2\bullet 0=\ldots=(p-1)\bullet 0=0,$$
$$1\bullet 1=1,$$
$$1\bullet\alpha=\alpha\bullet 1=\alpha,$$
$$\alpha^2=\alpha\bullet\alpha,$$
$$\alpha^3=\alpha\bullet\alpha\bullet\alpha$$
$$\vdots$$
$$\alpha^j=\alpha\bullet\alpha\bullet\bullet\bullet\bullet\alpha\quad (j\ times)$$
$$\vdots$$
$$\alpha^i\bullet\alpha^j=\alpha^j\bullet\alpha^i=\alpha^{i+j}$$

Now the following set of elements on which a multiplication operation "•" is defined:

$$F = \{0, 1, \alpha, \alpha^2, ..., \alpha^j, ...\}$$

Let $p(x)$ be a primitive polynomial of degree m over $GF(p)$, assume α be a new element such that $p(\alpha) = 0$. Since $p(x)$ divides $x^{p^m-1} + (p-1)$, then, $x^{p^m-1} + (p-1) = q(x)p(x)$.

If we replace x by α in the above equation, we obtain

$$\alpha^{p^m-1} + (p-1) = q(\alpha)p(\alpha).$$

Since $p(\alpha) = 0$, we have

$$\alpha^{p^m-1} + (p-1) = q(\alpha) \bullet 0.$$

If we regard $q(\alpha)$ as a polynomial of α over $GF(p)$, it follows that $q(\alpha) \bullet 0 = 0$. As a result, we obtain the following equality

$$\alpha^{p^m-1} + (p-1) = 0.$$

Adding 1 to both sides of the above equation (use modulo-p) result in the following equality

$$\alpha^{p^m-1} = 1.$$

Therefore, under the condition that $p(\alpha) = 0$, the set F becomes finite and contains the following elements

$$F^* = \left\{0, 1, \alpha, \alpha^2, ..., \alpha^{p^m-2}\right\}.$$

2.2. Irreducible Polynomial

A polynomial is said to be irreducible if it cannot be factored into nontrivial polynomials over the same field. Any irreducible polynomial over $GF(p)$ of degree m divides $x^{p^m-1} + (p-1)$.

For example, in the field of rational polynomials $Q[x]$ (*i.e.*, polynomials $f(x)$ with rational coefficients), $f(x)$ is said to be irreducible if there do not exist two non constant polynomials $g(x)$ and $h(x)$ in x with rational coefficients such that $f(x) = g(x)h(x)$.

Nagell [12] showed in the finite field $GF(2)$, $x^2 + x + 1$ is irreducible, but $x^2 + 1$ is not, since $(x+1)(x+1) = x^2 + 2x + 1 = x^2 + 1 \pmod 2$. That is, a polynomial is irreducible in $GF(p^m)$ if it does not factor over $GF(p^m)$. Otherwise it is reducible.

2.3. Primitive Root

There is at least one element in every Galois field, different powers of which give the different non-zero elements of the field, such an element is called primitive root of that field. The non-zero elements may be represented either as polynomials degree at most $(m-1)$ as we know that the power of primitive root x such that, $x^{p^m-1} + (p-1) = 0$.

In $GF(p^m)$, a nonzero element *is* said to be primitive if the order of x is $p-1$. The powers of a primitive element generate all the nonzero elements of $GF(p^m)$. x is the primitive root of $GF(p^m)$, if x satisfies the equation $x^{p^m-1} + (p-1) = 0$.

For example, in $GF(7)$, the equation $x^{7-1} + (7-1) = 0$ or, $x^6 + 6 = 0$.

The above equation satisfies for $x = 2$ *or* $3 \pmod 7$. But only for different powers of $x = 3$ under $\bmod 7$, which give the different non-zero elements of the field.

$3^1 = 3$, $3^2 = 2$, $3^3 = 6$, $3^4 = 4$, $3^5 = 5$, $3^6 = 1$. From the sequence we get $1, 2, 3, 4, 5, 6$. Therefore 3 is the primitive root of $GF(7)$.

List of Primitive roots of Galois fields of order p

Order of Field (*P*)	Primitive root *x*
3	2
5	2
7	3
11	2
13	2
17	3
19	2
23	5

2.4. Minimum Function

If the function $f(x)$ can be factorized with the help of $GF(p^m)$ then the function $f(x)$ is called the minimum function of $GF(p^m)$. If the minimum function is suitably chosen, the class of standard representative x will be a primitive element of $GF(p^m)$.

2.5. Calculating Minimum Function of *GF*(7)

We know the polynomial $x^{p^m-1} + (p-1)$ over $GF(p^m)$.

For $GF(7)$, polynomial reduced to $x^{7-1} + (7-1) = 0$ or, $x^6 + 6 = 0$.

To obtain the minimum function we divide $x^6 + 6$ by the least common multiple of all factors lies $x^d + 1$, where d is a divisor of $p^m - 1 = 7^1 - 1 = 6$.

Therefore, divide $x^6 + 6$ by $x+1$ and get a cyclotomic equation $6x^5 + x^4 + 6x^3 + x^2 + 6x + 1$. After factorized the cyclotomic equation, we get two minimum functions $x^4 + x^2 + 1$ and $6x + 1$.

This paper have been only considered the minimum function $x^4 + x^2 + 1$. Now replace x by α in the above equation and equating to zero and generating the different elements of $GF(7)$.

$\alpha^4 + \alpha^2 + 1 = 0$

Or, $\alpha^4 = 6\alpha^2 + 6$

Or, $1 = 6\alpha^4 + 6\alpha^2$

The elements are α, α^2, α^3, $\alpha^4 = 6\alpha^2 + 6$

$\alpha^5 = \alpha * \alpha^4 = \alpha \bullet (6\alpha^2 + 6) = 6\alpha^3 + 6\alpha$

$\alpha^6 = \alpha * \alpha^5 = \alpha \bullet (6\alpha^3 + 6\alpha) = 6\alpha^4 + 6\alpha^2 = 1$.

List of Primitive Polynomials for Selected Values of p and n

GF	f(x)
2^2	$x^2 + x + 1$
2^3	$x^3 + x + 1$
2^4	$x^4 + x + 1$
2^5	$x^5 + x^2 + 1$
3^2	$x^2 + x + 2$
3^3	$x^3 + 2x + 1$
5^2	$x^2 + x + 2$

3. Latin Square

A Latin square is an ancient puzzle where you try to figure out how many ways Latin letters can be arranged in a set number of rows and columns (a matrix); each symbol appears only once in each row and column. It's called a *Latin* square because it was developed based on Leonard Euler's works, which used Latin symbols. However, any letters can be used.

3.1. Orthogonal Latin Square

A Latin square arrangement is an arrangement of s symbols in s rows and s columns, such that every symbol occurs once in each row and each column. When two Latin squares of same order are superimposed on one another, in the resultant array if every ordered pair of symbols occurs exactly once, then the two Latin squares are said to be orthogonal.

3.2. Mutual Orthogonal Latin Square

If in a set of Latin squares, any two Latin squares are orthogonal then the set is called Mutually Orthogonal Latin Squares (MOLS) of order s.

Mutually orthogonal Latin squares of order 3

The following two Latin squares are orthogonal to each other.

A	B	C
C	A	B
B	C	A

α	β	γ
β	γ	α
γ	α	β

When the two squares are superimposed, we obtain the following Graeco-Latin square.

Aα	Bβ	Cγ
Cβ	Aγ	Bα
Bγ	Cα	Aβ

3.3. Graeco-Latin Square

Two $n \times n$ Latin squares are orthogonal to each other if each letter of the first square occurs in the same position as each letter of the second square exactly once. Such a pair is often called a Graeco-Latin square, because traditionally Latin letters are used for the first square and Greek letters for the second square.

3.4. Balanced Incomplete Block Design

Randomized block design and Latin square design are honored as complete block design where each block, row and column contains all the treatments under consideration. These complete blocks design as orthogonal designs are efficient having simple analysis. But for large number of treatments in experiments, it may be difficult or even impossible to obtain large size homogenous blocks to accommodate all the treatment in each block. These clumsy situations led to the experiments to use incomplete block design in which the number of experimental units per block is less than the number of treatments under consideration. In fact an incomplete block design is a design where block size is less than the number of treatments to be compared.

Thus a BIB design, an arrangement of v treatments in b blocks each of size $k\,(< v)$ such that

i) Each treatment occurs at most once in a block
ii) Each treatment occurs in exactly r blocks
iii) Each pair of treatments occurs together in exactly λ blocks.

Where, $v, b, k, r, \lambda \in Z^+$ and Z^+ is the set of integer number.

4. Result

Primitive elements of Galois field $GF(7)$ are 0, 1, α, α^2, α^3, $6(\alpha^2+1)$ and $6(\alpha^3+\alpha)$ with the minimum function $\alpha^4 + \alpha^2 + 1$, then the elements of $GF(7)$ can be obtained as follows

Table 1. *First two way additive table of the elements of GF(7).*

+	**0**	**1**	α	α^2	α^3		$6(\alpha^3+\alpha)$
0	0	1	α	α^2	α^3		$6(\alpha^3+\alpha)$
1	1	2	$\alpha+1$		α^3+1	$6\alpha^2$	$6(\alpha^3+\alpha)+1$
α	α	$\alpha+1$	2α	$\alpha^2+\alpha$	$\alpha^3+\alpha$	$6\alpha^2+\alpha+6$	$6\alpha^3$
α^2	α^2		$\alpha^2+\alpha$	$2\alpha^2$	$\alpha^3+\alpha^2$	6	$6\alpha^3+\alpha^2+6\alpha$
α^3	α^3	α^3+1	$\alpha^3+\alpha$	$\alpha^3+\alpha^2$	$2\alpha^3$	$\alpha^3+6(\alpha^2+1)$	6α
		$6\alpha^2$	$6\alpha^2+\alpha+6$	6	$\alpha^3+6(\alpha^2+1)$	$5\alpha^2+5$	$6(\alpha^3+\alpha^2+\alpha+1)$
$6(\alpha^3+\alpha)$	$6(\alpha^3+\alpha)$	$6(\alpha^3+\alpha)+1$	$6\alpha^3$	$6\alpha^3+\alpha^2+6\alpha$	6α	$6(\alpha^3+\alpha^2+\alpha+1)$	$5\alpha^3+5\alpha$

Substituting $\alpha = 3$ (3 is the primitive root of $GF(7)$) and Reduced Mod 7, we get the first Latin square.

Table 2. *First Latin square.*

0	**1**	**3**	**2**	**6**	**4**	**5**
1	2	4	3	0	5	6
3	4	6	5	2	0	1
2	3	5	4	1	6	0
6	0	2	1	5	3	4
4	5	0	6	3	1	2
5	6	1	0	4	2	3

Next, we construct the table of second summation; we get second summation of the element of $GF(7)$ multiply by α in the table of first summation principle column.

Table 3. *Second two way additive table of the elements of GF(7).*

+	**0**	**1**	α	α^2	α^3		$6(\alpha^3+\alpha)$
0	0	1	α	α^2	α^3		$6(\alpha^3+\alpha)$
α	α	$\alpha+1$	2α	$\alpha^2+\alpha$	$\alpha^3+\alpha$	$6\alpha^2+\alpha+6$	$6\alpha^3$
α^2	α^2		$\alpha^2+\alpha$	$2\alpha^2$	$\alpha^3+\alpha^2$	6	$6\alpha^3+\alpha^2+6\alpha$
α^3	α^3	α^3+1	$\alpha^3+\alpha$	$\alpha^3+\alpha^2$	$2\alpha^3$	$\alpha^3+6(\alpha^2+1)$	6α
		$6\alpha^2$	$6\alpha^2+\alpha+6$	6	$\alpha^3+6(\alpha^2+1)$	$5\alpha^2+5$	$6(\alpha^3+\alpha^2+\alpha+1)$
$6(\alpha^3+\alpha)$	$6(\alpha^3+\alpha)$	$6(\alpha^3+\alpha)+1$	$6\alpha^3$	$6\alpha^3+\alpha^2+6\alpha$	6α	$6(\alpha^3+\alpha^2+\alpha+1)$	$5\alpha^3+5\alpha$
1	1	2	$\alpha+1$		α^3+1	$6\alpha^2$	$6(\alpha^3+\alpha)+1$

Substituting $\alpha = 3$ (3 is the primitive root of $GF(7)$) and Reduced Mod 7, we get the second Latin square.

Table 4. *Second Latin square.*

0	**1**	**3**	**2**	**6**	**4**	**5**
3	4	6	5	2	0	1
2	3	5	4	1	6	0
6	0	2	1	5	3	4
4	5	0	6	3	1	2
5	6	1	0	4	2	3
1	2	4	3	0	5	6

Now, construct the table of third summation. We get third summation of the elements of $GF(7)$ multiply by α^2 in table of first summation principle column.

Table 5. *Third two way additive table of the elements of GF(7).*

+	**0**	**1**	α	α^2	α^3		$6(\alpha^3+\alpha)$
0	0	1	α	α^2	α^3		$6(\alpha^3+\alpha)$
α^2	α^2		$\alpha^2+\alpha$	$2\alpha^2$	$\alpha^3+\alpha^2$	6	$6\alpha^3+\alpha^2+6\alpha$
α^3	α^3	α^3+1	$\alpha^3+\alpha$	$\alpha^3+\alpha^2$	$2\alpha^3$	$\alpha^3+6(\alpha^2+1)$	6α
		$6\alpha^2$	$6\alpha^2+\alpha+6$	6	$\alpha^3+6(\alpha^2+1)$	$5\alpha^2+5$	$6(\alpha^3+\alpha^2+\alpha+1)$
$6(\alpha^3+\alpha)$	$6(\alpha^3+\alpha)$	$6(\alpha^3+\alpha)+1$	$6\alpha^3$	$6\alpha^3+\alpha^2+6\alpha$	6α	$6(\alpha^3+\alpha^2+\alpha+1)$	$5\alpha^3+5\alpha$
1	1	2	$\alpha+1$		α^3+1	$6\alpha^2$	$6(\alpha^3+\alpha)+1$
α	α	$\alpha+1$	2α	$\alpha^2+\alpha$	$\alpha^3+\alpha$	$6\alpha^2+\alpha+6$	$6\alpha^3$

Substituting $\alpha = 3$ (3 is the primitive root of $GF(7)$) and Reduced Mod 7, we get the third Latin square.

Table 6. *Third Latin square.*

0	**1**	**3**	**2**	**6**	**4**	**5**
2	3	5	4	1	6	0
6	0	2	1	5	3	4
4	5	0	6	3	1	2
5	6	1	0	4	2	3
1	2	4	3	0	5	6
3	4	6	5	2	0	1

Now, construct the table of fourth summation. We get fourth summation of the elements of $GF(7)$ multiply by α^3 in table of first summation principle column.

Table 7. *Fourth two way additive table of the elements of GF(7).*

+	**0**	**1**	α	α^2	α^3		$6(\alpha^3+\alpha)$
0	0	1	α	α^2	α^3		$6(\alpha^3+\alpha)$
α^3	α^3	α^3+1	$\alpha^3+\alpha$	$\alpha^3+\alpha^2$	$2\alpha^3$	$\alpha^3+6(\alpha^2+1)$	6α
		$6\alpha^2$	$6\alpha^2+\alpha+6$	6	$\alpha^3+6(\alpha^2+1)$	$5\alpha^2+5$	$6(\alpha^3+\alpha^2+\alpha+1)$
$6(\alpha^3+\alpha)$	$6(\alpha^3+\alpha)$	$6(\alpha^3+\alpha)+1$	$6\alpha^3$	$6\alpha^3+\alpha^2+6\alpha$	6α	$6(\alpha^3+\alpha^2+\alpha+1)$	$5\alpha^3+5\alpha$
1	1	2	$\alpha+1$		α^3+1	$6\alpha^2$	$6(\alpha^3+\alpha)+1$
α	α	$\alpha+1$	2α	$\alpha^2+\alpha$	$\alpha^3+\alpha$	$6\alpha^2+\alpha+6$	$6\alpha^3$
α^2	α^2		$\alpha^2+\alpha$	$2\alpha^2$	$\alpha^3+\alpha^2$	6	$6\alpha^3+\alpha^2+6\alpha$

Substituting $\alpha=3$ (3 is the primitive root of $GF(7)$) and Reduced Mod 7, we get the fourth Latin square.

Table 8. *Fourth Latin square.*

0	**1**	**3**	**2**	**6**	**4**	**5**
6	0	2	1	5	3	4
4	5	0	6	3	1	2
5	6	1	0	4	2	3
1	2	4	3	0	5	6
3	4	6	5	2	0	1
2	3	5	4	1	6	0

For seventh order Latin square, we get 6 mutual orthogonal Latin square (MOLS). Thus four Latin square have been obtained. We claim that four Latin square are MOLS. Therefore combined first, second, third and fourth Latin square. The four Latin square treatments occurs once and only once given below,

Table 9. *Construction of mutual orthogonal Latin square.*

0000	**1111**	**3333**	**2222**	**6666**	**4444**	**5555**
1326	2430	4652	3541	0215	5063	6104
3264	4305	6520	5416	2153	0631	1042
2645	3056	5201	4160	1534	6312	0423
6451	0562	2014	1603	5340	3125	4236
4513	5624	0146	6035	3402	1250	2361
5132	6243	1465	0354	4021	2506	3610

The aim of this paper is to show the construction of balanced incomplete block design, where treatment $v=7$, block $b=35$, block size $k=4$, replication $r=20$ and pair of each treatments $\lambda=10$. Moreover using these theory several combination such as

$$v=7,\ b=7,\ k=3,\ r=3,\ \lambda=1$$

$$v=7,\ b=14,\ k=3,\ r=6,\ \lambda=2$$

$$v=7,\ b=14,\ k=6,\ r=12,\ \lambda=10$$

$$v=7,\ b=21,\ k=2,\ r=6,\ \lambda=1$$

$$v=7,\ b=21,\ k=3,\ r=9,\ \lambda=3$$

$$v=7,\ b=35,\ k=6,\ r=30,\ \lambda=25$$

$$v=7,\ b=42,\ k=6,\ r=30,\ \lambda=25$$ etc.

5. Application

To construct BIBD, 4 mutual orthogonal Latin square of order 7 have been considered. Since block size is 35, therefore, choosing any 5 rows except first row from MOLS given in table 9. This paper has been taken last five rows of MOLS and treatment combination shown in the following table given below.

Table 10. *Construction of Balanced Incomplete Block Design.*

Block	Treatment Combinations				Block	Treatment Combinations			
01	3	2	6	4	19	5	3	4	0
02	4	3	0	5	20	3	1	2	5
03	6	5	2	0	21	4	2	3	6
04	5	4	1	6	22	4	5	1	3
05	2	1	5	3	23	5	6	2	4
06	0	6	3	1	24	0	1	4	6
07	1	0	4	2	25	6	0	3	5
08	2	6	4	5	26	3	4	0	2
09	3	0	5	6	27	1	2	5	0
10	5	2	0	1	28	2	3	6	1
11	4	1	6	0	29	5	1	3	2
12	1	5	3	4	30	6	2	4	3
13	6	3	1	2	31	1	4	6	5
14	0	4	2	3	32	0	3	5	4
15	6	4	5	1	33	4	0	2	1
16	0	5	6	2	34	2	5	0	6
17	2	0	1	4	35	3	6	1	0
18	1	6	0	3					

The above table $v = number\ of\ treatment = 7$, $b = number\ of\ block = 35$, $k = block\ size = 4$, $r = replication = 20$, $\lambda = pair\ of\ treatments = 10$.

Check the relation of BIBD

i) $vr = bk = 140$

ii) $\lambda(v-1) = r(k-1) \quad or, 10\times(7-1) = 20\times(4-1)$

iii) $b \geq v(Fisher\ Inequality),\ 35 \geq 7$.

The three parametric relations of BIBD are satisfied. Similarly construction of BIBD using MOLS of Galois field $GF(7^m)$or $GF(p^m)$can be constructed in the same way.

6. Conclusions

When the experimental unit are heterogeneous and heterogeneity is occurred among the blocks then randomized block design (RBD) is applied. In RBD number of treatment and block size are equal. When number of treatment is greater than number of plot then RBD is failure to analyze the design. In that situation incomplete block design (IBD) is applied and analyze is done by balanced incomplete block design (BIBD).

If the researchers want to know the impact of first beginner food, in that situation they apply two from four items of first beginner foods in six age groups of child and after certain period health status of Childs are observed. This problem can be solved by the theory of balanced incomplete block design and construction of BIBD is important issue. If the number of treatment is prime or prime power and block is the multiple of treatment then using MOLS, construct a BIBD.

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