American Journal of Engineering, Technology and Society
2017; 4(5): 74-78
http://www.openscienceonline.com/journal/ajets
ISSN: 2381-6171 (Print); ISSN: 2381-618X (Online)

Solving Fisher's Equation by Using Modified Variational Iteration Method

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To cite this article

Yaseen Ul Rehman, Memmona Yaqub, Qazi Mahmood Ul-Hassan, Kamran Ayub, Ayesha Siddiqa. Solving Fisher's Equation by Using Modified Variational Iteration Method. *American Journal of Engineering, Technology and Society.* Vol. 4, No. 5, 2017, pp. 74-78.

Received: April 14, 2017; **Accepted**: June 30, 2017; **Published**: August DD, 2017

Abstract

Fisher equation is nonlinear partial differential equation which is used in various biological, chemical processes in engineering and sciences. In this paper, we use the numerical technique, Variational iteration method and its some sub sequenced modification to solve the Fisher's equation. Lagrange multipliers for identification of optimal value of parameters in a functional are the basis of Variational iteration method. This method can be used to find possible approximate or series solution of problem. This paper also contains the numerical solution of Fisher equation. The Important purpose of this paper is to check the reliability of these techniques for the solution of Fisher equation as compared to other techniques.

Keywords

Fisher Equation, Variational Iteration Method, Maple 18, Series Solution

1. Introduction

It is observed that nonlinear partial differential equation arise in modeling of the various phenomena in engineering and science. One of the important equation is Fisher equation which has well defined applications in various biological and chemical processes and in engineering such as gene propagation [1-2], combustion [3], autocatalytic chemical reaction [4], and tissue engineering [5-6]. Fisher [1] proposed one dimensional non-linear parabolic "partial differential equation" and it is a reaction diffusion type of equation that examines the wave proliferation of a beneficial quality gene in a population. It can also be mentioned as the kinetic advantage rate of a beneficial gene to show the propagation of viral mutant in an unlimited long habitat. It is one of the simplest reaction diffusion equation. So the study of this type of partial differential equation becomes a relevant area of research.

In present era, many authors investigated accurate solution of non-linear partial differential equation ("Babolian and Dastani 2012", "Lee and Sakthivel 2010a, b, 2011a, b & 2012", "Wazwaz 2004 & 2006", "Abbasbandy and Shirzadi 2010", "Krisnangkura et al. 2012", "Honga and Lub 2012", "Kabir et al. 2011", "Jabbari et al. 2011", "Shi et al. 2012", "Parand and rad 2012" and "Taha and Noorani 2013") who had been taking interest in non-linear physical phenomena as well as in different fields of engineering and physics. Some useful techniques were also presents like that the sine-cosine method, exponential function method, Jacobielliptic function method, similarity reduction method, tanh-method, homogeneous-balance method, tanh-coth method first integral method and also various numerical method applied for the solution of Non-Linear Partial Differential Equation such as variational iteration method, homotopy perturbation method finite volume method and so on. Many nonlinear phenomena are used to describe the reaction-diffusion equations or coupled equations [7-9] in biochemical and biological process. The simplest equation which explain the diffusion as well as for the presence of non-linear source term [10] is the Fisher equation.

And In this paper, we use some numerical technique, Variational Iteration Method and its some sub sequenced modification to solve the Fisher's equation.

We consider the Fisher's equation of the given form.

$$w_t = w_{xx} + w(1-w), \tag{1}$$

With initial condition $w(x,0) = \beta$.

Where $w_t = \frac{\partial u}{\partial t}, w_{xx} = \frac{\partial^2 u}{\partial u^2}$.

It is no secret to the researcher in the field of nonlinear partial differential equations, that the solution of this class of equations is not easy. So we find that many researchers have done and are still great efforts to find methods to solve this type of equations. These efforts resulted into formation of Variational Homotopy Peturbation Method, In short VHPM. The variational iteration method (VIM) that was established by J H He [11-12] and homotopy perturbation method which was also developed by He in 1998 [13-16]. VHPM is applied by many researchers to solve nonlinear and linear problems see [17-20].

A model for the propagation of mutantgene was proposed by Fisher [21]. The model also donated the density w of an advantageous. Chemical kinetics and Population dynamics use this equation. Moreover, this equation is used in flame propagation, neurophysiology, logistic growth models [21] and autocatalytic chemical reactions. Series solution of nonlinear differential equation is obtained by using an iterative formula which was introduced by Ji-Huan He [23-25].

2. Variational Iteration Method (VIM)

The variational iteration method handles linear and nonlinear problems in a straightforward manner. Unlike the Adomian decomposition method where we determine distinct components of the exact solution, the variational iteration method gives rapidly convergent successive approximations of the exact solution if such a closed form solution exists.

The standard ith order nonlinear Fredholm integro-differential equation is of the form

$$w^{(i)}(x) = f(x) + \int_0^1 K(x,t)F\big(u(t)\big)dt, \tag{2}$$

Where $w^{(i)}(x) = \frac{d^i w}{dx^i}$, and $F(u(x))$ is a nonlinear function of $w(x)$. The initial conditions should be prescribed for the complete determination of the exact solution.

The correction functional for the nonlinear integro-differential equation (2) is

$$w_{n+1}(x) = w_n(x) + \int_0^1 \lambda(t)\left(w_n^i(t) - f(t) - \int_0^t K(t,r)F(\widetilde{w_n}(r))dr\right)dt \tag{3}$$

To apply this method in an effective way, we should follow two essential steps:

(i) It is required first to determine the Lagrange multiplier λ that can be identified optimally via integration by parts and by using a restricted variation. The Lagrange multiplier λ may be a constant or a function.

(ii) Having λ determined, an iteration formula, without restricted variation, should be used for the determination of the successive approximations $w_{n+1}(x)$, $n \geq 0$ of the solution $w(x)$. The zeroth approximation w_0 can be any selective function. However, the initial values are preferably used for the selective zeroth approximation w_0. Consequently, the solution is given by

$$w(x) = \lim_{n\to\infty} w_n(x). \tag{4}$$

We consider the following nonlinear partial differential equation to represent the basic concept of VIM.

$$Lw(x,t) + Rw(x,t) + Nw(x,t) = g(x,t), w(x,0) = f(x), \tag{5}$$

Where $L \equiv \frac{\partial}{\partial x}, R$ a linear operator which has partial derivatives with respect to $x, Nw(x,t)$ is a nonlinear term and $g(x,t)$ is an inhomogeneous term.

We can derive the following iterative formula while keeping in view the VIM [23-25].

$$w_{n+1}(x,t) = w_n(x,t) + \int_0^t \lambda\{Lw_n + R\widetilde{w}_n + N\widetilde{w}_n - g\}\,d\tau \tag{6}$$

Where λ is a general language multiplier [26] that can be recognized optimally through variational theory, supposed $\widetilde{w}_n$ is as restricted variation [23], i.e. $\delta\,\widetilde{w}_n = 0$.

The following stationary conditions are obtained by calculating the variation with respect to w_n.

$$\lambda'(\tau) = 0, \tag{7}$$

$$1 + \lambda(\tau)|_{\tau=t} = 0, \tag{8}$$

The language multiplier can be identified as $\lambda = -1$.

Placing the identified Language multiplier into equation (6), gives the following formula.

$$w_{n+1}(x,t) = w_n(x,t) - \int_0^t \{Lw_n + Rw_n + Nw_n - g\}\,d\tau \tag{9}$$

The 2nd term on right is known as "Correction term". Equation (9) can be solved iteratively using $w_0(x,t) = f(x)$ As an initial approximation.

The variational iteration method (VIM) will be illustrated by studying the following examples.

3. Implementation of VIM

In this part, Considering the Fisher equation of the following form

$$w_t = w_{xx} + w(1-w), \tag{10}$$

With the initial condition $w(x,0) = \beta$,

To solve the equation (2) By using VIM, Putting the values of

$$Lw = w_{n_t}, Rw_n = -w_{n_{xx}}, Nw_n = -w_n(1-w_n),$$

And

$$g = 0,$$

in equation (6)

And get the given iteration formula

$$w_{n+1}(x,t) = w_n(x,t) - \int_0^t \{w_{n_t} - w_{n_{xx}} - w_n(1-w_n)\}\, d\tau \quad (11)$$

With the initial approximation $w(x,0) = \beta$.

The iterative scheme (3) will give the many successive approximations and obtained the exact solution at the limit of the resulting successive approximations

Using the iterative scheme (3) and Maple 18 software we get

$$w_1(x,t) = \beta + \beta(1-\beta)t, \quad (12)$$

$$w_2(x,t) = \beta + \beta(1-\beta)t + \beta(1-\beta)(1-2\beta)\frac{t^2}{2!} + \left[-\beta^2(1-\beta)^2\frac{t^3}{3!}\right], \quad (13)$$

$$w_3(x,t) = \beta + \beta(1-\beta)t + \beta(1-\beta)(1-2\beta)\frac{t^2}{2!} + \beta(1-\beta)(1-6\beta+6\beta^2)\frac{t^3}{3!} + \left[-\beta^2(1-2\beta)(1-\beta)^2\frac{t^4}{3} - \beta^2(1-\beta)^2(3-20\beta+20\beta^2)\frac{t^5}{60} + \beta^3(1-2\beta)(1-\beta)^3\frac{t^6}{60} - \beta^4(1-\beta)^4\frac{t^7}{63}\right], \quad (14)$$

and so on. The closed form of the solution is given as

$$w(x,t) = \beta + \beta(1-\beta)t + \beta(1-\beta)(1-2\beta)\frac{t^2}{2!} + \beta(1-\beta)(1-6\beta+6\beta^2)\frac{t^3}{3!} + \cdots w(x,t) = \frac{\beta e^t}{1-\beta+\beta e^t}, \quad (15)$$

which is satisfied with the results in [27].

The comparison between the 7th iterative of variational iteration method (VIM) and the exact solution shown in figure 1.

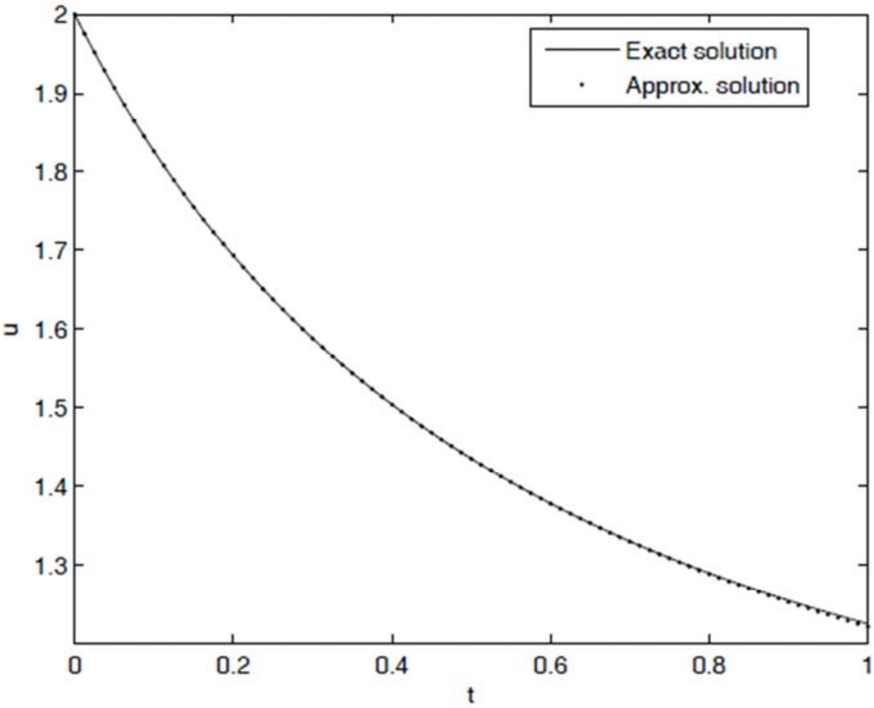

Figure 1. *Represent the exact solution and the 7th iterate of variational iteration method (VIM) for (β = 2).*

VIM by using He's polynomials

In this section, we apply the variational iteration method using He's polynomials (VIMHP)

Considering Fisher equation of given form

$$w_t - w_{xx} - 6w(1-w) = 0, \quad (16)$$

Subject to the initial conditions

$$w(x,t) = \frac{1}{(1+e^x)^2} \quad (17)$$

Where the correctional function is following

$$w_{n+1}(x,t) = w_n(x,t) + \int_0^t \lambda\left(\frac{\partial w_n(x,s)}{\partial s} - \frac{\partial^2 \widetilde{w}_n(x,s)}{\partial x^2} - 6\widetilde{w}_n(x,s)\left(1-\widetilde{w}_n(x,s)\right)\right)ds, \tag{18}$$

Where the above correctional function is stationary and the Lagrange multiplier can be identified as $\lambda(s)=-1$, consequently, So, we obtain the following iteration formula.

$$w_{n+1}(x,t) = w_n(x,t) - \int_0^t \left(\frac{\partial w_n(x,s)}{\partial s} - \frac{\partial^2 w_n(x,s)}{\partial x^2} - 6w_n(x,s)\left(1-w_n(x,s)\right)\right)ds \tag{19}$$

Applying the variational iteration method using He's polynomials (VIMHP), we get

$$w_0 + pw_1 + p^2w_2 + \cdots = w_0(x,t) - p\int_0^t\left(\frac{\partial w_0(x,s)}{\partial s} + p\frac{\partial w_1(x,s)}{\partial s} + p^2\frac{\partial w_2(x,s)}{\partial s} + \cdots\right)ds + p\int_0^t\left(\left(\frac{\partial^2 w_0(x,s)}{\partial x^2} + p\frac{\partial^2 w_1(x,s)}{\partial x^2} + \cdots\right) + \left((w_0 + pw_1 + \cdots)\left(1-(w_0+pw_1+\cdots)\right)\right)\right)ds. \tag{20}$$

Comparing the co-efficient of same powers of p, then we obtained the following approximation

$$p^{(0)}: w(x,t) = \frac{1}{(1+e^x)^2}, \tag{21}$$

$$p^{(1)}: w_1(x,t) = \frac{1+e^x(1+10t)}{(1+e^x)^3}, \tag{22}$$

$$p^{(2)}: w_2(x,t) = \frac{1}{(1+e^x)^6}\left(25e^x(1+e^x)^2(-1+2e^x)t^2 - 200e^{2x}t^3 + (1+e^x)^3(1+e^x(1+10t))\right), \tag{23}$$

$$p^{(3)}: w_3(x,t) = \frac{25e^x}{3(1+e^x)^6}\left((5-6e^x-15e^{2x}+20e^{3x})t^3\right) - \frac{50e^{2x}}{(1-e^x)^8}\left(-17+5e^x+52e^{2x}t^4\right) + \frac{150e^{2x}}{(1+e^x)^9}\left((5-47e^x+20e^{3x})t^5\right) + \frac{10000e^{3x}(-1+2e^x)t^6}{(1+e^x)^{10}} - \frac{240000e^{4x}t^7}{7(1+e^x)^{12}} + \frac{1}{(1+e^x)^6}\left(25e^x(1+e^x(1+e^x)^2(-1+2e^x)t^2 - 200e^{2x}t^3 + (1+e^x(1+10t))\right), \tag{24}$$

$$\vdots$$

So, the series solution is given as

$$w(x,t) = \frac{25e^x}{3(1+e^x)^6}\left((5-6e^x-15e^{2x}+20e^{3x})t^3\right) - \frac{50e^{2x}}{(1-e^x)^8}\left(-17+5e^x+52e^{2x}t^4\right) + \frac{150e^{2x}}{(1+e^x)^9}\left((5-47e^x+20e^{3x})t^5\right) + \frac{10000e^{3x}(-1+2e^x)t^6}{(1+e^x)^{10}} - \frac{240000e^{4x}t^7}{7(1+e^x)^{12}} + \frac{1}{(1+e^x)^6}\left(25e^x(1+e^x(1+e^x)^2(-1+2e^x)t^2 - 200e^{2x}t^3 + (1+e^x(1+10t))\right) + \cdots \tag{25}$$

And the close form of above series solution is

$$w(x,t) = \frac{1}{(1+\exp(x-5t))^2}. \tag{26}$$

Table 1. *Numerical results for Fisher's equation.*

	t = 0.2		t = 0.4	
X	***E (ADM)**	***E(VIMHP)**	***E (ADM)**	***E(VIMHP)**
0	07.22002E-03	02.69962E-03	05.75298E-02	05.01844E-02
0.2	09.89049E-03	02.15566E-03	01.6115E-01	05.27127E-02
0.4	01.09765E-02	01.136097E-03	01.39113E-01	04.12072E-02
0.6	01.04039 E-02	07.38299E-04	01.51579E-01	02.25459E-02
0.8	08.50732 E-03	05.73101E-04	01.43529E-01	05.28693E-03
01	05.87222 E-03	09.07727E-04	01.19333E-01	04.23672E-03

$*\ Error = Exect\ Solution - Series\ Solution$

Table 1 shows the errors between Adomian' decomposition method (ADM) and the VIMHP.

4. Conclusion

In this paper, variational iteration method has been used to get the exact solution of Fisher's equation. The supremacy of VIM has been shown by the graphical and tabular results. We use the MAPLE software to calculate the series solution obtained from variational iteration method in our work. The obtained results are very encouraging. It is concluded that variational iteration method is an efficient technique for nonlinear differential equations.

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