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# The Reciprocal Hall Effect, CPT Symmetry and the Second Law

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## Abstract

Onsager reciprocals are extended to systems that include a magnetic field. A new phenomenon, the reciprocal Hall effect, is proposed: when a magnetic field is applied parallel to a surface and an electric field is applied perpendicular to the surface, a current is spontaneously generated along the surface, perpendicular to both fields. This phenomenon is shown to produce no second law violation when particles are homogeneous and indistinguishable, in compliance with the H-theorem which assumes homogeneity and indistinguishability. However, second law violations do arise when the reciprocal Hall Effect is implemented in heterogeneous systems in which particles can be distinguished by their physical attributes such as their statistics. Such systems fall outside the coverage of the H-Theorem and therefore are not restricted by the second law. One must then choose between full Onsager reciprocity, CPT symmetry and the second law. They cannot all be correct.

## Keywords

Onsager Reciprocal, Second Law, Homogeneity, Indistinguishability, Entropy, Statistical Symmetry, Hall Effect, H-theorem

## 1. Introduction

*"Nothing in physics seems so hopeful to as the idea that it is possible for a theory to have a high degree of symmetry was hidden from us in everyday life. The physicist's task is to find this deeper symmetry.*" Steven Weinberg. [1]

This paper extends Onsager reciprocals to phenomena, such as the Hall Effect, that include a magnetic field. In his paper, Onsager limits the domain of applicability of his reciprocals to systems complying with the second law and detailed balance. He disclaims systems involving magnetic forces and Coriolis forces which could lead to such violations. He states [2] (page 411):

"…how could equilibrium be maintained? Besides a certain number of transitions balancing each other directly according the to the scheme

$$\begin{aligned} A &\rightleftarrows B \\ B &\rightleftarrows C \\ C &\rightleftarrows A \end{aligned} \tag{1}$$

we should have additional transitions taking place around the cycle [3]:

$$A \to B \to C \to A \tag{2}$$

Onsager continues:

*"Now the idea of an equilibrium maintained by a mechanism like equation* (2) *whether entirely or only in part, is not in harmony with our notion that molecular mechanisms has much in common with the mechanics of ordinary conservative dynamical systems. Barring certain exceptional cases [Coriolis forces, external magnetic fields (and permanent magnets)] which can readily be recognized and sorted out, the dynamical laws of familiar conservative systems are always reversible, that means: if the velocities of all the particles present are reversed simultaneously, the particles will trace their former paths, reversing the entire succession of configurations…."*

Yet he acknowledges experimentally proven phenomena but excludes them from having reciprocals which would cause a second law violation. These phenomena include the Righi-Leduc effect [2 page 425]:

"When magnetic forces and Coriolis forces destroy the reversibility of macroscopic motion, we must expect that the microscopic motion will fare no better… We may expect that these relations will break down in cases where magnetic or Coriolis forces are acting, and they do… However if a transverse magnetic field is applied the temperature gradient will have a component in the third direction perpendicular to flow and field. The direction of the temperature gradient is rotated with respect to the heat flow, about an axis parallel to the magnetic field."

The phenomena also include the Hall effect [2, page 425]

"More familiar than the Righi-Leduc effect is the Hall effect. When a constant current is flowing through a metallic conductor a transverse magnetic field causes an e0.f. perpendicular to both."

Onsager requires time reversibility for his reciprocals:

"One consequence of this principle of dynamic reversibility is the condition that when a molecule changes a certain number of times per second from configuration A to the configuration B the direct reverse transition $B \rightarrow A$ must take place equally often."

He barely recognizes but time charge symmetrical systems:

"in the presence of a magnetic field the principle of microscopic reversibility may be applied in a modified form. The entire motion may be reversed by reversing the magnetic field together with the velocities of all particles composing a dynamical system."

and he completely ignores full CPT symmetry. His reciprocals are restricted to time symmetric reversible systems.

One is faced with three choices:

Either CPT symmetry is broken.

Or Onsager's reciprocals are broken if extended to full CPT symmetry.

Or the second law as currently understood, is broken.

Onsager's solution is to pick the second choice. In other words, Onsager fails to take seriously his own reciprocals when they violate the second law. His disclaimer is ad hoc, not based on any justification inherent to the reciprocal relations or mechanisms themselves. If one is to explore the limitations of this law, one needs to consider reciprocals with full CPT symmetry and see where they lead, rather than a-priori assume second law limitations as Onsager does. This paper shows that one needs to make a choice between the validity of reciprocals, CPT symmetry or the second law. They cannot all be correct.

## 2. The Reciprocal Hall Effect

A reciprocal Hall effect is proposed in which, in the presence of perpendicular electric and magnetic fields, a current is spontaneously produced perpendicular to both fields [4].

This effect can be implemented in a specially configured thermo-ionic vacuum chamber shown in cross-section in Figure 1. The chamber includes an insulator walls traversed by cesium or rubidium strips. The chamber is heated to a temperature of several 1000°C, sufficient to allow electrons to overcome the surface potential and induce thermo-ionic emission from the strips. The reason for the strip configuration will become clearer further down in the paper.

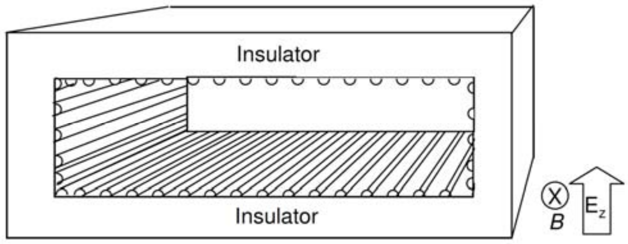


***Figure 1.*** *Reciprocal Hall Effect Chamber. The suggested experiment includes a thermo-ionic chamber comprising an insulator floor, traversed with cesium or rubidium strips.*

The foregoing analysis is semi-classical. For the sake of simplicity, electrons are assumed to have high temperature and low density. They are degenerate and possess the statistical property of the surface that emits them. This assumption will be relaxed and shown to be immaterial to the conclusion of this paper. Consider an electron with charge $q$, mass $m$ and subjected to a magnetic field $B_y$ parallel to the bottom surface of the chamber as shown in Figure 2.

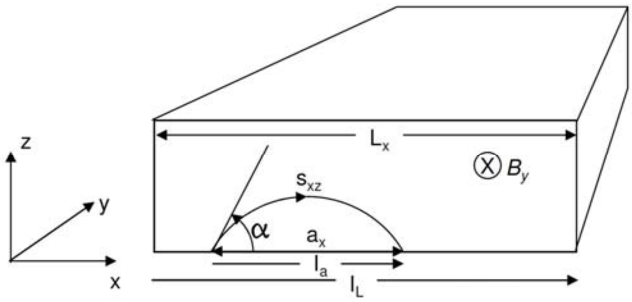


***Figure 2.*** *Reciprocal Hall Effect. The thermo-ionic chamber of Figure 1 is subjected to a magnetic field parallel to the floor of the chamber. Emitted electrons follow interrupted clockwise orbits producing a left current on the bottom surface and a right current on the top surface.*

As an electron collides with the surface, it is thermalized and acquires the distribution of the atoms at the surface which is assumed to be maxwellian. Therefore, immediately after re-emission, that is before the electron has any time to travel, its velocity is half-maxwellian. However, after some time has elapsed, the magnetic field *skews the distribution away from the normal* to the layer. As the electron returns to the surface, it has a *pronouncedly biased* half-maxwellian distribution.

The question being asked is how much current $\boldsymbol{I_x}$ is produced along the length $\boldsymbol{L_x}$ of the layer by this electron as it bounces near the surface.

The re-emitted electron shall at first be assumed to travel only in a vertical plane (xz) perpendicular to the field $\boldsymbol{B_y}$, with velocity $\boldsymbol{v_{xz}}$ along a circular arc $\boldsymbol{S_{xz}}$ beginning and ending at the surface of the layer. If the magnetic field points into the plane of the drawing, the electron must always travel clockwise. On the bottom surface, the endpoint is *always* to the right of the starting point, generating a surface drift current to the left and vice versa, on the top surface, the endpoint is *always* to the left, and the surface drift current is to the right.

The electron following the clockwise arc $\boldsymbol{S_{xz}}$ produces a net drift current along the chord $\boldsymbol{a_x}$ on the surface of the layer. Let the electron leave the surface at a tangential angle $\boldsymbol{\alpha}$. The length of the arc is then $\boldsymbol{S_{xz} = 2\alpha r}$ where $\boldsymbol{\alpha}$ can range from 0 for a zero-length arc to $\boldsymbol{\pi}$ for a full orbit. The travel time from the starting point to the ending point along the arc is $\boldsymbol{t = 2\alpha r / v_{xz}}$ which is also the equivalent travel time of the charge projected along the chord $\boldsymbol{a_x}$. The surface drift current along $\boldsymbol{a_x}$ is $\boldsymbol{I_a = q/t = qv_{xz} / 2\alpha r}$. The current $\boldsymbol{I_L(\alpha)}$ over the whole length $\boldsymbol{L_x}$ of the layer needs to be scaled accordingly by $\boldsymbol{a_x / L_x}$:

$$I_L(\alpha) = \frac{a_x}{L_x}\frac{qv_{xz}}{2\alpha r} \tag{3}$$

Since $a_x = 2r\sin(\alpha)$:

$$I_L(\alpha) = \frac{qv_{xz}}{L_x}\frac{\sin(\alpha)}{\alpha} \tag{4}$$

Now assuming an electron density $\boldsymbol{n}$ at the surface, the current becomes

$$I_L(\alpha) = \frac{nq}{2L_x}\frac{\sin(\alpha)}{\alpha}v_{xz} \tag{5}$$

As mentioned above, this current represents only the contribution of electrons moving in a plane perpendicular to the magnetic field. The total surface drift current $\boldsymbol{I_x}$ is obtained by integrating $\boldsymbol{I_L(\alpha)}$ with a Maxwell-Boltzmann distribution expressed in polar coordinates. The integration is taken over half a sphere to get the half-maxwellian distribution which is correct at the starting point, immediately after re-emission:

$$I_x = \int_{-\pi/2}^{\pi/2}\int_0^{\pi/2}\int_0^{\infty} I_x(\alpha)\, f_{MB}(v)\,dv\,d\theta\,d\varphi \tag{6}$$

Substituting equation (5) into equation (6) yields:

$$I_x = \int_{-\pi/2}^{\pi/2}\int_0^{\pi/2}\int_0^{\infty} \frac{nq}{2L_x}\frac{\sin(\alpha)}{\alpha} v f_{MB}(v)\,dv\,d\theta\,d\varphi \tag{7}$$

Since $\boldsymbol{v}$ is constant over the domain of integration for $\boldsymbol{\theta}$ and for $\boldsymbol{\varphi}$, we can treat each integral in equation (6) separately. Since $\boldsymbol{\theta = \pi/2 - \alpha}$, we can evaluate

$$\int_{-\pi/2}^{\pi/2}\frac{\sin(\alpha)}{\alpha}d\theta = -\int_0^{\pi}\frac{\sin(\alpha)}{\alpha}d\alpha = Si(\pi) = 1.85 \quad \text{and}$$

$$\int_0^{\pi/2} d\varphi = \frac{\pi}{2} \tag{8}$$

Hence

$$I_x = -1.85\frac{nq}{2L_x}\frac{\pi}{2}\int_0^{\infty} v f_{MB}(v)\,dv \tag{9}$$

The Maxwell-Boltzmann distribution with a potential energy term is obtained from equation (11) in [5]. Removing the potential energy term the average velocity $\boldsymbol{<v>}$ is:

$$< v > = \int_0^{\infty} v f_{MB}(v)\,dv = \int_0^{\infty}\left(\frac{m}{2\pi k_B T}\right)^{3/2}\exp\left(\frac{-mv^2}{2k_B T}\right)4\pi v^3 dv \tag{10}$$

and using

$$\int_0^{\infty} x\exp(-x)dx = \Gamma(2) = 1 \tag{11}$$

from [6, 7] we can solve the integral in equation (10)

$$< v > = \int_0^{\infty} v f_{MB}(v)\,dv = \left(\frac{8}{\pi}\right)^{1/2}\left(\frac{k_B T}{m}\right)^{1/2} \tag{12}$$

Combining equations (9) and (12) produces

$$I_x = -1.85\frac{nq}{2L_x}\frac{\pi}{2}\left(\frac{8}{\pi}\right)^{1/2}\left(\frac{k_B T}{m}\right)^{1/2} \tag{13}$$

Rearranging we get

$$I_x = -1.85\left(\frac{\pi}{2}\right)^{1/2}\frac{nq}{L_x}\left(\frac{k_B T}{m}\right)^{1/2} \tag{14}$$

As shown in Figure 3, the current flowing on the bottom surface is moving left, and the one on the top surface is moving right. The currents have equal magnitudes but opposite directions resulting in *a zero-net observable current* if one was to connect an ammeter across the edges of the layer.

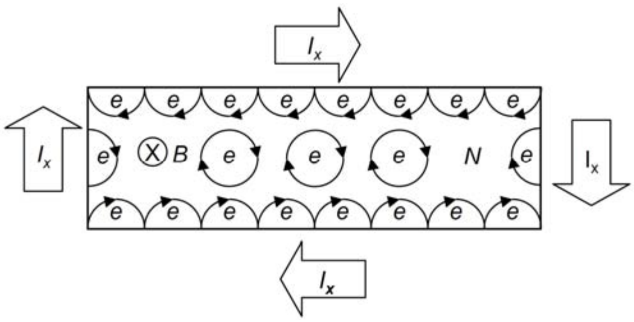


***Figure 3.*** *A magnetic field alone is insufficient to produce an observable current. The clockwise movement of electrons produces a right current on the bottom surface and left current on the top surface. The currents connect to each other as they wrap around the edges of the layer. They are equal and opposite, and not directly measurable from the edge.*

Let us now apply a voltage $\boldsymbol{V_z}$ as required by the proposed reciprocal Hall effect, between the insulated capacitor plates positioned on either side of the chamber as shown in Figure 4. This voltage produces an electric field $\boldsymbol{E_z}$ across the chamber.

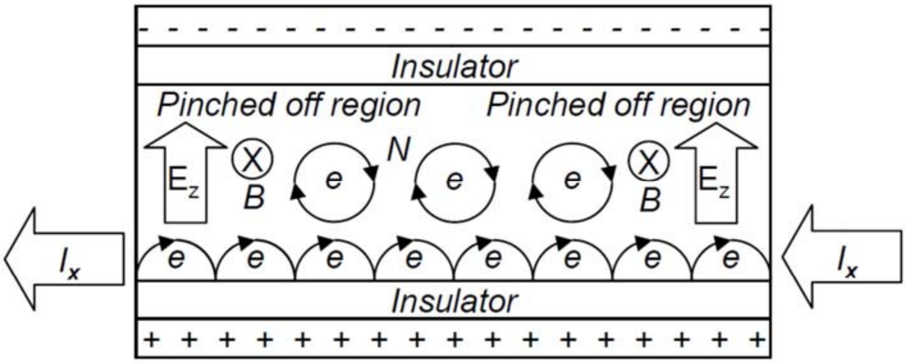


***Figure 4. A*** *Combination of magnetic field and electric field produces an observable current. The electric field pinches off the right current at the top surface leaving an observable left current at the bottom surface.*

If the magnetic force is significantly larger than the electric force ($\boldsymbol{qv}_{xz} \times \boldsymbol{B}_y >> \boldsymbol{qE}_z$) we can assume that the electrons follow mostly circular orbits and that the calculations above are correct except that the electric field modifies the concentration of electrons by shifting them from the top of the chamber to the bottom. The electron density shall be assumed to be low to moderate and the chamber thin enough that the number of electrons shifted by the electric field is insufficient to generate a significant space charge that would cancel the electric field $\boldsymbol{E}_z$.

The shift in electrons results in a difference $\Delta n$ in the number of electrons between the two sides of the chamber. Let the electron density be low and the temperature be high to minimize electron-electron interaction and maximize degeneracy. Under these conditions, the thermal interaction between the electrons and the chamber's surfaces defines the electrons' statistics as maxwellian. The change in the electron concentration between the top and bottom of the chamber is:

$$\Delta n = n\left(1 - \exp\left(\frac{-qV_z}{k_B T}\right)\right) \tag{15}$$

The net current for the top and bottom surfaces of the chamber is obtained by combining equations (13) and (15) yielding

$$\Delta I_x = -1.85\left(\frac{\pi}{2}\right)^{1/2} \frac{nq}{L_x} \left(\frac{k_B T}{m}\right)^{1/2} \left(1 - \exp\left(\frac{-qV_z}{k_B T}\right)\right) \tag{16}$$

indicating that a measurable current can be spontaneously generated.

The current described by equation (16) is driven by thermal energy extracted from the material when the electrons are thermalized by their collision with the floor of the chamber.

A graphing program [8] has been written to plot the velocity of electrical carriers due to the reciprocal Hall Effect, for at any given elevation and magnetic field as described in Figure 5.

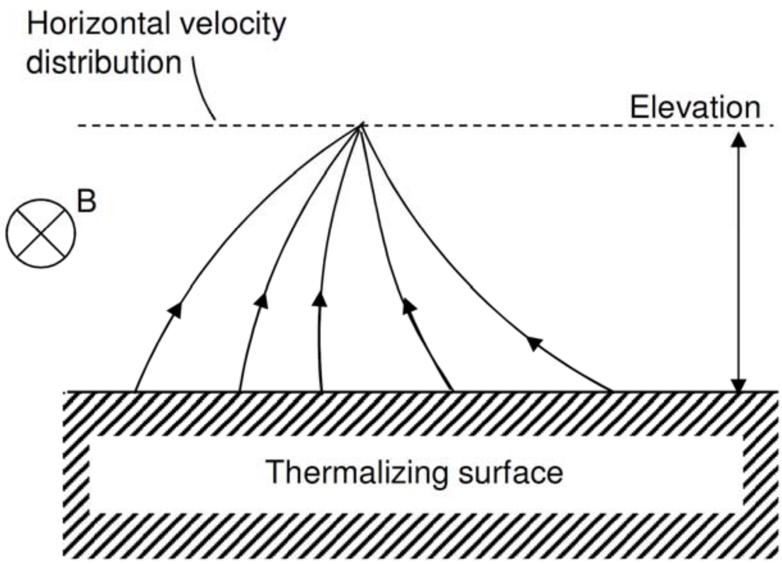


***Figure 5.*** *Graphing Program. The velocity distribution is calculated as a function of elevation and magnetic field, for electrons after they have been thermalized by a surface.*

The control panel and display of the reciprocal Hall Effect calculator is shown in Figure 6 for an elevation of 300nm and a field of 0.2 Tesla.

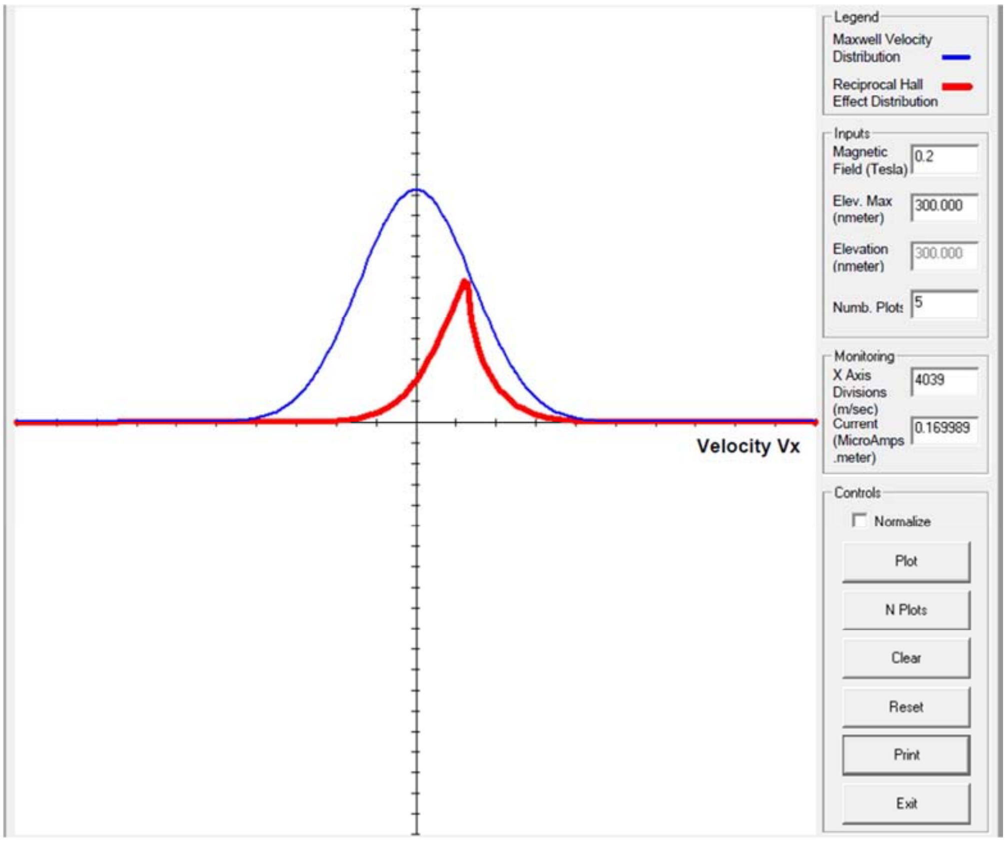


***Figure 6.*** *Reciprocal Hall Effect Calculator. This program shows in red the electron velocity distribution due to the reciprocal Hall Effect at 300 nm above a surface. The magnetic field is assumed to be 0.2 Tesla parallel to the surface. The blue curve provides is reference showing the distribution at zero elevation. The horizontal axis shows 4039 m/sec per division.*

Figure 7 (a)-(f) shows distributions for different elevations.

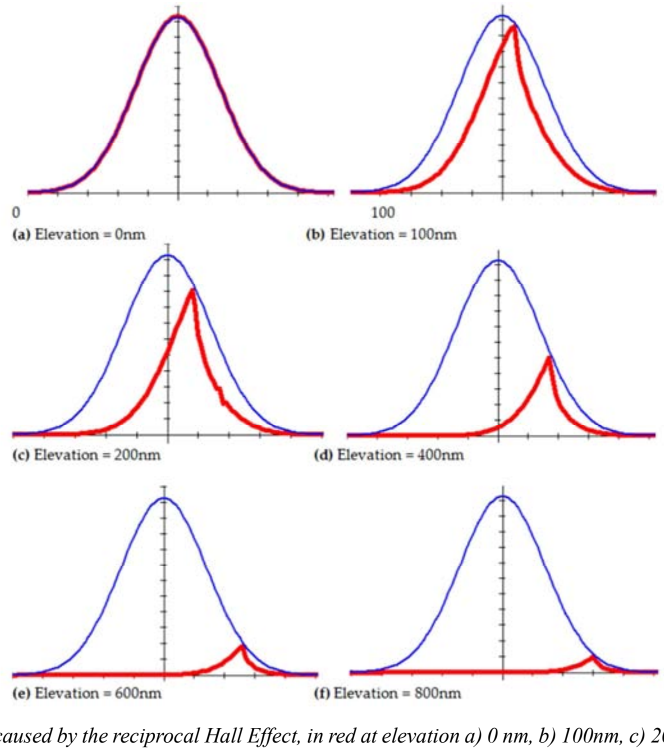


***Figure 7.*** *Electron velocity distribution caused by the reciprocal Hall Effect, in red at elevation a) 0 nm, b) 100nm, c) 200 nm, d) 400nm, e) 600nm and f) 800 nm above a surface for a magnetic field of 0.2 Tesla parallel to the surface. The blue curve provides a reference showing the distribution at zero elevation. The horizontal axis shows 4039 m/sec per division.*

The rational for the walls of the chamber to be insulators (see Figure 1) is now clear. This configuration only allows the reciprocal Hall Effect current at the surface and perpendicular to the strips. Without insulator walls, this surface current would be short circuited and masked by current in the bulk of the chamber's walls. The walls need to be insulators and the strips must be perpendicular to the flow of surface current.

The configuration of Figure 1 promotes current on a surface and is reminiscent of a topological insulator. In the anomalous quantum Hall effect, a surface flow mechanism [9] resembling the one depicted in Figure 2, produces steps in the Hall resistance accompanied by zero or near zero longitudinal resistance. This is due to carriers' ½ spin causing any reflection of the wave function by an obstacle to be eliminated by destructive interference.

A low (room) temperature reciprocal Hall Effect could be implemented by thermoelectrics. These materials owe their remarkable properties to the relatively low coupling between the electrical carriers – electrons or holes – and the supporting crystal lattice. Thermoelectric efficiency is described by the figure of merit $ZT = \sigma(S_b)^2T/\kappa$ where $\sigma$ is the carriers' electrical conductivity, $\kappa$ is the phonons' thermal conductivity, $S_b$ is the Seebeck coefficient, and $T$ is the temperature. Electrical carriers can be considered to be in a gas phase and the material, effectively transparent to carriers.

Accordingly, a thin thermoelectric slab could operate as an analog to the empty space within the chamber of Figure 1, allowing carriers to follow orbits interrupted at the surface of the slab. The material should be lightly doped and operate depletion mode to avoid the buildup of space charges that would cancel the electric field.

Experimental and other supporting evidence for the reciprocal Hall effect, more particularly the chiral property of electrical carriers near a surface, in the presence of a magnetic field are well documented [9-14].

## 3. Discussion

The movement of electrons causes a voltage to develop across the device in Figure 4. Can this voltage be exploited to produce power in violation of the second law? Given the large body of evidence collected over nearly two centuries, such a violation would be extraordinary.

Before answering whether the law is violated, one should be reminded that steady-state electrical potentials, *per se,* as the one produced by the reciprocal Hall Effect in Figure 4 is not unique. For example, a semiconductor junction produces a built-in potential but this potential is not accessible for power production. Connecting leads – the reverse path - across the junction – the forward path - does not produce any current because the contact potential between the leads and the semiconductor material exactly cancels the built-in potential. *The electrical carriers in the lead are identical with the carriers in the junction and they are subjected to the same electric field.* Since the field is scalar, a loop summation of potential around the electrical loop is zero. Therefore, the second law is preserved.

What happens if the electrical loop in Figure 4 is closed by means of a reverse path in which carriers have exactly the same biased statistics as in the forward path as illustrated in Figure 8?

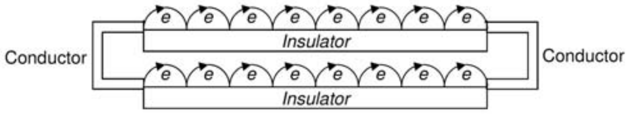


***Figure 8.*** *Homogeneous forward and reverse paths cannot produce a current. No current flows when electrical carriers in the reverse path are homogeneous with, indistinguishable from, and have the same statistics as the carriers in the forward path.*

No current flows because the reverse path potential exactly cancels the forward path potential in the same manner as in the semiconductor junction example. This result is consistent with the H-theorem [15-19] which provides a theoretical justification for the second law. This theorem states that entropy can never decrease in an isolated system with the important assumptions that the system is homogeneous and its constituent particles, indistinguishable. The semiconductor junction example and the reciprocal example in Figure 9 comply with these assumptions. The carriers in the forward path and in the reverse path are identical in every way including their statistical distribution.

Consider the system in Figure 8 in which the reciprocal Hall Effect voltage is tapped by conventional metal electrical leads unaffected by the reciprocal Hall Effect. The leads are connected to a motor or electrical resistance. The statistics of the carriers are biased in the chamber by the surface/fields interaction, but are not in the leads. Therefore, this system does not comply with the assumptions of homogeneity and indistinguishability required by the H-Theorem and there is no theoretical reason why it should not violate the second law.

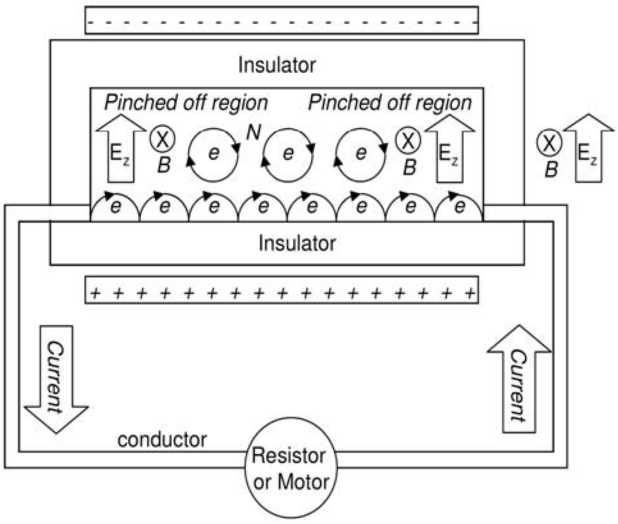


***Figure 9.*** *The electrical carriers in the leads to the motor or resistor have statistics different from those in the chamber. The system falls outside the scope of the H-theorem. The forward path generates a reciprocal Hall effect current, and the reverse path utilizes this current.*

Electrical carriers in the reciprocal Hall Effect discussed in the previous section have a Fermi-Dirac distribution which has been approximated by a Maxwell-Boltzmann distribution. This was done for the sake of simplicity by considering degenerate carriers at high temperature and low density. However, this approximation is not necessary. One can envision that a Fermi-Dirac distribution would also have been biased away from the normal to the surface by the interaction of the surface and the fields.

A system conforms with the H-Theorem if it complies with the homogeneity and indistinguishability assumptions including statistical uniformity or symmetry. A heterogeneous system in which this statistical symmetry is broken falls outside the coverage of the H-Theorem. Such systems include particles that follow different statistics such as Maxwell-Boltzmann, Fermi-Dirac, Bose Einstein, or distributions biased by surface/field interaction. An example is provided by the author in [5]. A heterogeneous system comprised of fermions (electrically charged heat carrier in a thermoelectric material) and bosons (neutral heat phonons in thermal connectors to the thermoelectric) can produce a usable temperature difference output when a field is applied across the material. This effect is distinct from well-known thermoelectric effects (e.g., Seebeck) as it does not require any electrical input. The effect has been observed in the lab [20, 21] but reported as unexplained laboratory measurement error. The Faraday isolator is another instance of systems falling outside the coverage of the H-Theorem because of the influence of a magnetic field. This system is described by the author in [22]. The author discusses non-transitive thermodynamics that fall outside the coverage of the H-Theorem in [23].

## 4. Conclusion

Onsager was right in his belief that extending his reciprocals to full CPT symmetry, for example by using magnetic fields, would violate the second law. He limits the scope of his reciprocals without providing any justification beyond saying that such extended reciprocals would result in a violation. This paper shows that extending Onsager reciprocals to full CPT symmetry is not theoretically forbidden. For example, the reciprocal Hall Effect falls outside the coverage of the H-Theorem.

The paper also shows that compliance with the second law depends on the homogeneity and indistinguishability of a system's constituent particles, and on the statistical symmetry of its components.

This statistical symmetry is broken in the reciprocal Hall Effect in which perpendicular electric and magnetic fields induce carriers on a surface to follow a distribution biased away from the normal to the surface. A spontaneous voltage is produced perpendicular to both fields and parallel to the surface. This voltage becomes accessible for energy production if a current loop is formed, in which the reverse path comprises electrical carriers statistically heterogeneous from those in the forward path.

Onsager's ad hoc partial rejection of his own reciprocals needs to be re-examined. Therefore, for heterogeneous systems, one is forced to accept only two of the following: reciprocals, CPT symmetry or the second law. They cannot all be correct.

## Acknowledgments

I would like to thank my wife, Penny, for her unwavering support.

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[3] Note: the original equation by Onsager $A \rightarrow B \leftarrow C \rightarrow A$ is assumed to have a typographical error.

[4] Note: There are two reciprocal Hall effects. The most interesting one is discussed at length in this paper. The second is trivial. It is the generation of a perturbation in the primary magnetic field in the presence of a perpendicular electric field and a perpendicular current. This perturbation is produced by the shifting of the current's path by the electric field.

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