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Thermal Instability in a Rivlin-Ericksen Elastico-Viscous Nanofluid in a Porous Medium: A Revised Model

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Abstract

Thermal instability in a horizontal layer of Rivlin-Ericksen elastico-viscous nanofluid in porous medium is considered for more realistic boundary conditions. A linear stability analysis based upon perturbation method and normal mode technique is used to find solution of the fluid layer. The onset criterion for stationary convection is derived analytically to study the stability characteristics. The effects of the Lewis number, modified diffusivity ration, nanoparticle Rayleigh number and porosity parameter on the stability of the system are investigated analytically.

Keywords

Nanofluid, Rivlin-Ericksen Fluid, Thermal Instability, Viscosity, Visco-Elasticity, Perturbation Method

1. Introduction

Thermal instability in a layer nanofluid saturated in porous medium based upon a model presented by Buongiorno [1] is now regarded as a classical problem due to its wide range of applications in geothermal reservoirs, agricultural product storage, enhanced oil recovery, packed-bed catalytic reactors and the pollutant transport in underground. There are large numbers of papers on the thermal instability of nanofluid based upon the Buongiorno's model on the assumption that one could control the value of the nanoparticles fraction at the boundary in the same way as the temperature there could be controlled. The choice of the boundary conditions imposed on nanoparticles fraction is somewhat arbitrary, it could be argued that zero-flux for nanoparticles volume fraction is more realistic.

Recently Nield and Kuznetsov [2] and Chand and Rana [3-4], Chand et al. [5], Rand and Chand [6], studied the thermal instability in a horizontal layer of nanofluid by taking normal component of the nanoparticle flux zero at boundary which is more physically realistic. In this paper we revised the Chand and Rana [7] by considering more realistic boundary condition on the volume fraction of nanoparticles i.e. the normal flux of volume fraction of nanoparticles is zero on the boundaries.

With the importance in various applications mentioned Chand and Rana [8-9], our main aim is to study the thermal instability of Rivlin-Ericksen elastico-viscous nanofluid in porous medium for more realistic boundary conditions.

2. Mathematical Formulations

Consider an infinite horizontal layer of Rivlin-Ericksen elastico-viscous nanofluid of thickness'd' bounded by plane z = 0 and z = d heated from below in a porous medium of porosity ε and medium permeability k_1. Each boundary wall is assumed to be impermeable and perfectly thermal conducting. Fluid layer is acted upon by gravity force g(0,0,-g). The normal component of the nanoparticles flux has to vanish at an impermeable boundary and the temperature T is taken to be T_0 at z = 0 and T_1 at z = d, ($T_0 > T_1$). The reference scale for temperature and nanoparticles fraction is taken to be T_1 and φ_0 respectively. Thermophysical properties of the nanofluid are constant for the analytical formulation but these properties are not constant and strongly depend upon volume fraction of the nanoparticles.

The governing equations for Rivlin-Ericksen elastico-viscous nanofluid in a porous medium (Chandrasekhar [10], Nield and Kuznetsov [2], Chand and Rana [6]) are

$$\nabla \cdot q = 0, \tag{1}$$

$$0 = -\nabla p + \left(\varphi\rho_p + (1-\varphi)\left\{\rho_{f_0}\left(1-\alpha\left(T-T_1\right)\right)\right\}\right)g - \frac{1}{k_1}\left(\mu + \mu'\frac{\partial}{\partial t}\right)q, \tag{2}$$

$$(\rho c)_m \frac{\partial T}{\partial t} + (\rho c)_f\, q\cdot\nabla T = k_m \nabla^2 T + \varepsilon(\rho c)_p\left(D_B \nabla\varphi\cdot\nabla T + \frac{D_T}{T_1}\nabla T\cdot\nabla T\right), \tag{3}$$

$$\frac{\partial\varphi}{\partial t} + \frac{1}{\varepsilon} q\cdot\nabla\varphi = D_B\nabla^2\varphi + \frac{D_T}{T_1}\nabla^2 T, \tag{4}$$

where q is the velocity of nanofluid, p is the pressure, ρ_0 is the density of nanofluid at lower layer, φ is the volume fraction of the nanoparticles, ρ_p density of nanoparticles, T is the temperature, α is coefficient of thermal expansion, g is acceleration due to gravity, μ is viscosity, μ' kinematic visco-elasticity, ε porosity and k_1 medium permeability, $(\rho c)_m$ is effective heat capacity of fluid in a porous medium, $(\rho c)_p$ is heat capacity of nanoparticles and k_m is effective thermal conductivity of the porous medium, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient of the nanoparticles.

We assume that the temperature is constant and nanoparticles flux is zero on the boundaries. Thus boundary conditions (Nield and Kuznetsov[2]) are

$$w=0,\quad T=T_0,\quad D_B\frac{\partial\phi}{\partial z}+\frac{D_T}{T_1}\frac{\partial T}{\partial z}=0 \text{ at } z=0 \text{ and}$$

$$w=0,\quad T=T_1\quad D_B\frac{\partial\phi}{\partial z}+\frac{D_T}{T_1}\frac{\partial T}{\partial z}=0 \text{ at } z=d. \tag{5}$$

Introducing non-dimensional variables as $(x',y',z') = \left(\frac{x,y,z}{d}\right)$, $q'(u',v',w') = q\left(\frac{u,v,w}{\kappa}\right)d$, $t' = \frac{\kappa}{\sigma d^2}t$, $p' = \frac{k_1}{\mu\kappa}p$, $\varphi' = \frac{(\varphi-\varphi_0)}{\varphi_0}$, $T' = \frac{(T-T_1)}{(T_0-T_1)}$, where $\sigma = \frac{(\rho c_p)_m}{(\rho c_p)_f}$ is thermal capacity ratio, $\kappa = \frac{k_m}{(\varrho c)_m}$ is thermal diffusivity of the fluid.

Equations (1)-(5), in non-dimensional form can be written as

$$\nabla\cdot q' = 0, \tag{6}$$

$$0 = -\nabla p' - \left(1+F\frac{\partial}{\partial t'}\right)q' - Rm\hat{e}_z + RaT'\hat{e}_z - Rn\varphi'\hat{e}_z, \tag{7}$$

$$\frac{\partial T'}{\partial t'} + q'\cdot\nabla T' = \nabla^2 T' + \frac{N_B}{Le}\nabla\varphi'\cdot\nabla T' + \frac{N_A N_B}{Le}\nabla T'\cdot\nabla T', \tag{8}$$

$$\frac{1}{\sigma}\frac{\partial\varphi'}{\partial t'} + \frac{1}{\varepsilon}q'\cdot\nabla\varphi' = \frac{1}{Le}\nabla^2\varphi' + \frac{N_A}{Le}\nabla^2 T', \tag{9}$$

Boundary conditions are

$$w' = T' = 0,\quad \frac{\partial\phi'}{\partial z'} + N_A\frac{\partial T'}{\partial z'} = 0 \text{ at } z=0 \text{ and}$$

$$w' = T' = 0,\quad \frac{\partial\phi'}{\partial z'} + N_A\frac{\partial T'}{\partial z'} = 0 \text{ at } z=1. \tag{10}$$

where non-dimensional parameters are

$Le = \frac{\kappa}{D_B}$ is the Lewis number,

$F = \frac{\mu'\kappa}{\mu\sigma d^2}$ is the Kinematic visco-elasticity parameter,

$Ra = \frac{\rho_0 g\alpha\Delta T k_1 d}{\mu\kappa}$ is the thermal Rayleigh number,

$Rm = \frac{\left(\rho_p\varphi_0 + \rho(1-\varphi_0)\right)gdk_1}{\mu\kappa}$ is the density Rayleigh number,

$Rn = \frac{(\rho_p-\rho)(\varphi_1-\varphi_0)gdk_1}{\mu\kappa}$ is the nanoparticle Rayleigh number,

$N_A = \frac{D_T\Delta T}{D_B T_1\varphi_0}$ is the modified diffusivity ratio,

$N_B = \frac{(\rho c)_p\varphi_0}{(\rho c)_f}$ is the modified particle-density increment.

2.1. Basic Solutions

The basic state of the nanofluid is assumed to be time independent and is described by $q'(u,v,w)=0,\ p'=p'(z),\ T'=T_b(z),\ \varphi'=\varphi_b(z)$.

Equations (6) – (9) using boundary conditions (10) give solution as

$$T_b = 1 - z,\quad \varphi_b = \phi_0 + N_A z. \tag{11}$$

Where φ_0 is reference value for nanoparticles volume fraction.

The basic solution for temperature is same as the solution obtained by Nield and Kuznetsov [2], Chand and Rana [3].

2.2. Perturbation Solutions

To study the stability of the system, we superimposed infinitesimal perturbations on the basic state, which are of the forms

$$q' = 0 + q'', T' = T_b + T'', \varphi' = \varphi_b + \varphi'', p' = p_b + p'' \quad \text{with } T_b = 1\text{-}z, \varphi_b = \phi_0 + N_A z. \tag{12}$$

$$\nabla \cdot q = 0, \tag{13}$$

$$0 = -\nabla p - \left(1 + F\frac{\partial}{\partial t'}\right) q + Ra T \hat{e}_z - Rn \varphi \hat{e}_z, \tag{14}$$

$$\frac{\partial T}{\partial t} - w = \nabla^2 T + \frac{N_B}{Le}\left(N_A \frac{\partial T}{\partial z} - \frac{\partial \varphi}{\partial z}\right) - \frac{2N_A N_B}{Le}\frac{\partial T}{\partial z}, \tag{15}$$

$$\frac{1}{\sigma}\frac{\partial \varphi}{\partial t} + \frac{N_A}{\varepsilon} w = \frac{1}{Le}\nabla^2 \varphi + \frac{N_A}{Le}\nabla^2 T, \tag{16}$$

Boundary conditions are

$$w = T = 0, \ \frac{\partial \phi}{\partial z} + N_A \frac{\partial T}{\partial z} = 0 \ \text{at} \ z = 0 \ \text{and} \ z = 1. \tag{17}$$

[Dashes ('') are dropped are for simplicity]
Eliminating 'p' from equation (14), we get

$$\left(1 + \frac{\partial}{\partial t} F\right)\nabla^2 w = Ra \nabla_H^2 T - Rn \nabla_H^2 \varphi, \tag{18}$$

where ∇_H^2, is two-dimensional Laplacian operator.

3. Normal Modes

Analyzing the disturbances into the normal modes and assuming that the perturbed quantities are of the form

$$[w, T, \varphi] = [W(z), \Theta(z), \Phi(z)] \exp(ik_x x + ik_y y + nt), \tag{19}$$

where k_x, k_y are wave numbers in x and y direction and n is the growth rate of disturbances.
Using equation (19), equations (18), (15) and (16) become

$$(1 + nF)(D^2 - a^2)W + a^2 Ra \Theta - a^2 Rn \Phi = 0, \tag{20}$$

$$\frac{W}{\varepsilon} - \frac{N_A}{Le}(D^2 - a^2)\Theta - \left(\frac{1}{Le}(D^2 - a^2) - \frac{n}{\sigma}\right)\Phi = 0, \tag{21}$$

$$W + \left(D^2 - a^2 - n + \frac{N_A}{Le}D - \frac{2N_A N_B}{Le}D\right)\Theta - \frac{N_B}{Le}D\Phi = 0. \tag{22}$$

Where $D \equiv \frac{d}{dz}$ and $a = \sqrt{k_x^2 + k_y^2}$ is dimensionless the resultant wave number.
The boundary conditions are

$$W = 0, \Theta = 0, D\Phi + N_A D\Theta = 0 \ \text{at} \ z = 0, 1. \tag{23}$$

4. Method of Solution

The Galerkin weighted residuals method is used to obtain an approximate solution to the system of equations (20) – (22) with the corresponding boundary conditions (23). In this method, the test functions are the same as the base (trial) functions. Accordingly W, Θ and Φ are taken as

$$W = \sum_{p=1}^{N} A_p W_p, \ \Theta = \sum_{p=1}^{N} B_p \Theta_p, \Phi = \sum_{p=1}^{N} C_p \Phi_p. \tag{24}$$

Where A_p, B_p and C_p are unknown coefficients, p =1, 2, 3,...N and the base functions W_p, Θ_p and Φ_p are assumed in the following form

$$W_p = \Theta_p = z^p - z^{p+1}, \ \Phi_1 = N_A(z^2 - z) \ \text{and} \ \Phi_p = \frac{1}{2}N_A z^2, p = 2, 3, 4...... \tag{25}$$

such that W_p, Θ_p and Φ_p satisfy the corresponding boundary conditions. Using expression for W, Θ and Φ in equations (20) – (22) and multiplying first equation by W_p second equation by Θ_p and third by Φ_p and integrating in the limits from zero to unity, we obtain a set of 3N linear homogeneous equations in 3N unknown A_p, B_p and C_p; p = 1, 2, 3,...N. For existing of non trivial solution, the vanishing of the determinant of coefficients produces the characteristics equation of the system in term of Rayleigh number Ra.

5. Linear Stability Analysis

We confine ourselves to the one- term Galerkin approximation. The eigenvalue equation of the problem is given as

$$Ra = \frac{1}{a^2}\left((a^2+10)\left(\left((a^2+10)+n\right)(1+nF)\right)\right) - \frac{(a^2+10)N_A + \frac{Le}{\varepsilon}\left((a^2+10)+n\right)}{(a^2+10) + \frac{nLe}{\sigma}} Rn. \tag{26}$$

For neutral stability real part of n is zero. On putting n = iω, (where ω is real and is dimensionless frequency of oscillation) in equation (26), we have

$$\mathrm{Ra}=\frac{1}{\mathrm{a}^2}\left(\left(a^2+10\right)\left(\left(\left(a^2+10\right)+i\omega\right)\left(1+i\omega F\right)\right)\right)-\frac{\left(a^2+10\right)N_A+\frac{Le}{\varepsilon}\left(\left(a^2+10\right)+i\omega\right)}{\left(a^2+10\right)+\frac{i\omega\mathrm{Le}}{\sigma}}\mathrm{Rn}. \quad (27)$$

Since oscillatory convection has been ruled out, because of the absence of two opposing buoyancy forces, we need to consider the case of stationary convection (ω = 0). For stationary convection (ω = 0), equation (27) reduces to

$$\mathrm{Ra}=\frac{\left(\mathrm{a}^2+10\right)^2}{a^2}-\left(N_A+\frac{Le}{\varepsilon}\right)Rn. \quad (28)$$

It is clear from equation (28) that the kinematic visco-elasticity parameter F vanishes with n and the Rivlin-Ericksen elastico-viscous fluid behaves like an ordinary Newtonian fluid. It is the good agreement of the result obtained Nield and Kuznetsov [2].

Also stationary Rayleigh number Ra is a depend upon dimensionless wave number 'a', Lewis number Le, modified diffusivity ratio N_A, porosity ε and nanoparticles Rayleigh number Rn, but independent of modified particle -density increment N_B, and density Rayleigh number Rm. The interweaving behaviors' of Brownian motion and thermoporesis of nanoparticles evidently does not change the critical size of the Bénard cell at the onset of instability. As such, the critical size is not a function of any thermophysical properties of nanofluid.

The minimum value of first term of RHS of equation (28) attained when wave number $\mathrm{a}\approx\sqrt{10}$ and minimum value is ≈ 40.

Thus critical Rayleigh number Ra_c for steady onset is given by

$$\mathrm{Ra}_c=40-\left(N_A+\frac{Le}{\varepsilon}\right)Rn.$$

In the absence of nanoparticles (Rn = Le = N_A = 0) i.e. for ordinary fluid, critical Rayleigh number Ra_c for steady onset is given by $\mathrm{Ra}=40$, which is approximately equal to well-known result that the critical Rayleigh number $Ra_c = 4\pi^2$

6. Results and Discussions

The expression for stationary thermal Rayleigh number is given in equation (28) are computed as a function of Lewis number, concentration Rayleigh number, modified diffusivity ratio and porosity parameter and the parameters considered are in the range $10^2 \le Ra \le 10^5$ (thermal Rayleigh number), $1 \le N_A \le 10$ (modified diffusivity ratio), $10^2 \le Le \le 10^4$ (Lewis number), $Rn > 0$ (nanoparticles Rayleigh number) and $10^{-3} \le \varepsilon \le 10^{-2}$ (porosity parameter)[Chand and Rana [3,9]].

We examine the behavior of $\frac{\partial \mathrm{Ra}}{\partial \mathrm{L}e}$, $\frac{\partial \mathrm{Ra}}{\partial \mathrm{N}_A}$, $\frac{\partial \mathrm{Ra}}{\partial Rn}$ and $\frac{\partial \mathrm{Ra}}{\partial \varepsilon}$ analytically to study the effect of Lewis number, modified diffusivity ratio, nanoparticle Rayleigh number and porosity parameter.

From equation (28), we have

i. $\frac{\partial \mathrm{Ra}}{\partial \mathrm{Le}}<0$, which imply that for stationary convection Lewis number has destabilizing effect on the fluid layer.

ii. $\frac{\partial \mathrm{Ra}}{\partial \mathrm{N}_A}<0$, which mean that modified diffusivity ratio destabilizes the fluid layer for stationary convection.

iii. $\frac{\partial \mathrm{Ra}}{\partial Rn}<0$, which imply that for stationary convection nanoparticle Rayleigh number has destabilizing effect on fluid layer.

iv. $\frac{\partial \mathrm{Ra}}{\partial \varepsilon}>0$, which imply that for stationary convection porosity parameter had stabilizing effect on the fluid layer.

These results are good agreement with results derived by Chand and Rana [3-4].

7. Conclusions

Thermal instability in a horizontal layer of Rivlin-Erickson elastico-viscous nanofluid is studied by taking normal component of the nanoparticle flux zero at boundary which is more physically realistic. We draw following conclusions:

(i). The critical cell size is not a function of any thermo physical properties of nanofluid.

(ii). The Lewis number, modified diffusivity ratio and nanoparticle Rayleigh number have destabilizing effect while porosity parameter has stabilizing effects on the stationary convection.

(iii). Oscillatory convection has been ruled out, because of the absence of two opposing buoyancy forces.

(iv). For stationary convection Rivlin-Ericksen elastico-viscous nanofluid fluid behaves like an ordinary Newtonian fluid.

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