# An Attempt to Describe Quantum Interference on Two Slits in Classical Terms 

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#### Abstract

An idea of interpretation of quantum interference in classical terms is presented. Classical propagation of an electron through a slit in a perfectly conducting screen is considered. The change of the electron trajectory under influence of a nearby slit in the same screen is evaluated. The goal of this study is to see whether the influence of the second slit on the electron trajectory can be interpreted as interference in quantum mechanics.


## Keywords

Quantum Mechanics, Interference, Electron Diffraction, Hidden Parameters

## 1. Introduction

In quantum mechanics diffraction of a particle on 2 slits in a screen is described as interference of a wave, and the particle is supposed to go simultaneously through both slits. Such a picture does not look realistic, and those, who worry about interpretation of quantum mechanics, try to devise a more realistic picture. First attempt was made by de Broglie [1], who interpreted wave function as a field of a point particle. Next step was done by Bohm [2]. He introduced the so-called quantum potential. However, the Bohmian mechanics [3-6] is only a special way to solve the Schrödinger equation. The de Broglie's approach was considered also in [7]. There was made an attempt to explain with it an anomaly of the ultracold neutrons storage in closed vessels [8]. In [9] it was proposed a pure classical approach to interference without use of the Schrödinger equation. The idea of this approach is the following. Suppose that a particle is a classical point-like object, and its wave function is some field like the Coulomb one in the case of electrons. Motion of the particle is defined by the Newton equation

$$
\begin{equation*}
\frac{d^{2}}{d t^{2}} \mathrm{r}(t)=\mathrm{F}(|\psi(\mathrm{r}, t)|) \tag{1}
\end{equation*}
$$

in which the force depends on the field at the position of the
particle. It is its own field reflected from surrounding bodies. In order to find the force it is necessary to solve the field equation

$$
\begin{equation*}
\frac{\Delta}{2} \psi(\mathrm{r}, t)+i \frac{\partial}{\partial t} \psi(\mathrm{r}, t)=b \delta(\mathrm{r}-\mathrm{r}(t)), \tag{2}
\end{equation*}
$$

where $\mathrm{r}(t)$ is the trajectory defined by the first equation, and solution of Eq. (2) must be found with account of boundary conditions on the surrounding objects. For instance, if a particle (imagine a classical electron) moves through a slit on the screen $S_{t}$, as shown in Fig. 1, its trajectory is determined by action of its own field reflected from the screen, and reflection depends on boundary conditions at the screen surface. If there is only one slit in the screen $S_{t}$, the particle will strike the detecting screen $S_{0}$ at some point. If there appears another slit in the screen $S_{t}$, the boundary conditions will change; therefore the field and trajectory of the particle also change. Then the particle will strike the detecting screen $S_{0}$ at another point, as shown in Fig. 1. So, there is an interference between two slits, or perturbation of the particle trajectory going through one slit created by simple presence of the nearby second slit. The wave function, or wave field, defined by Eq. (2) can be in the form of the Coulomb field for classical electrons, or de Broglie's wave packet [1], which for free particles with speed v looks like

$$
\begin{equation*}
\psi(\mathrm{r}, t) \equiv \psi(s, \mathrm{v}, \mathrm{r}, t)=\sqrt{\frac{s}{2 \pi}} \frac{e^{-s \mathrm{r}-\mathrm{v} \mid} \mid}{|\mathrm{r}-\mathrm{v} t|} e^{i \mathrm{rr}-i \omega t} \tag{3}
\end{equation*}
$$

where $s$ is the wave packet width, $\omega=\left(v^{2}-s^{2}\right) / 2$, and here we use the units where $\hbar=m=1$. The wave packet (3) is normalized to unity:

$$
\int_{-\infty}^{+\infty}|\psi(s, \mathrm{v}, \mathrm{r}, t)|^{2} d^{3} r=1
$$

and satisfies the equation

$$
\begin{equation*}
\left(\frac{\Delta}{2}+i \frac{\partial}{\partial t}\right) \psi(s, \mathrm{v}, \mathrm{r}, t)=-\sqrt{\frac{s}{2 \pi}} 2 \pi \delta(\mathrm{r}-\mathrm{v} t) e^{i\left(\mathrm{v}^{2}+s^{2}\right) t / 2} . \tag{4}
\end{equation*}
$$



Figure 1. An experiment with classical electron going through the upper slit in the screen St. Due to the electron's field interaction with the St, its trajectory after the screen depends on whether the other slit is opened or not. It is an illustration of interference of two slits in classical physics.

The Fourier expansion of the wave packet is

$$
\begin{equation*}
\psi(s, \mathrm{v}, \mathrm{r}, t)=\sqrt{\frac{s}{2 \pi}} \frac{4 \pi}{(2 \pi)^{3}} e^{i(\mathrm{vr}-\omega t)} \int d^{3} p \frac{\exp (i \mathrm{p}[\mathrm{r}-\mathrm{v} t])}{p^{2}+s^{2}} . \tag{5}
\end{equation*}
$$

So, if we can find reflection of the plane waves in this expansion from the screen, and their scattering on the particle, we can find trajectory of the particle.

Solution of the system of equations (1) and (2) is a formidable task, and the quantum mechanics is an ingenious theory, which replaced these two equations by the single linear, Schrödinger equation, but this replacement removed causality and introduced probabilities into the theory.

We attempted to solve the system of equations (1) and (2) in the simplest case of an electron with its Coulomb field moving through a slit in an ideally conducting screen. We succeeded to find the interference. The result is not so much interesting, however, we decided to publish it in hope that some scientists with better mathematical background will be able to improve our approach and find a more interesting result.

## 2. Formulation of the Problem

Let's consider the simplest case of an electron motion started at the point $s$ (see Fig. 2). At the point $r$ it is attracted to the screen, and the attraction force is determined by the image force. It means that, if the distance to the screen $S_{t}$ at the point $r$ is equal to $x$, then the electron with charge $q$ is attracted to the screen with the force $F=q^{2} / 4 x^{2}$, as if the screen was infinite without a slit. In the case when the electron is located opposite to the slit, no force acts on it, and the electron moves with the constant speed, which it had at the edge of the slit. The Newton equation of motion in this case can be solved analytically. The motion along the screen (coordinate $y$ ) is uniform, i.e. $y=y_{0}+v_{y} t$. The motion towards the screen is defined by the equation

$$
\begin{equation*}
\ddot{x}=\frac{1}{m} \frac{q^{2}}{(2 x)^{2}}=\frac{\xi}{x^{2}}, \tag{6}
\end{equation*}
$$

where $\xi=q^{2} / 4 m$. Multiplication of both parts of Eq. (6) by $d x / d t$ reduces it to

$$
\begin{equation*}
\frac{d}{d t} \dot{x}^{2}=-\frac{d}{d t} \frac{2 \xi}{x} \tag{7}
\end{equation*}
$$

From which it follows

$$
\begin{equation*}
\dot{x}^{2}=-\frac{2 \xi}{x}+C \tag{8}
\end{equation*}
$$

where constant $C$ is defined at $t=0$, when $x=-l$, and $l$ is the initial distance between the electron and the screen. Therefore

$$
\begin{equation*}
C=v_{0 x}^{2}-\frac{2 \xi}{l} \tag{9}
\end{equation*}
$$

Substitution of (9) into (8) gives

$$
\begin{equation*}
\dot{x}=\sqrt{C-\frac{2 \xi}{x}} \tag{10}
\end{equation*}
$$



Figure 2. An electron with charge $q$ at distance $x$ from the perfectly conducting screen is attracted to it with the image force $q^{2} / x^{2}$.
which after substitution

$$
\begin{equation*}
x=-L s h^{2} \theta, L=\frac{2 \xi}{C} \tag{11}
\end{equation*}
$$

is reduced to

$$
\begin{equation*}
\int_{\theta_{0}}^{\theta} d z \operatorname{sh}^{2} z=-\delta t \tag{12}
\end{equation*}
$$

where

$$
\begin{equation*}
\delta=\frac{C^{3 / 2}}{2 \xi}, \theta_{0}=\operatorname{ash} \sqrt{\frac{l}{L}} . \tag{13}
\end{equation*}
$$

The integral $I$ at the left side of (12) can be calculated by parts
$I=\int d \mathrm{chzsh} z=\operatorname{ch} z \operatorname{sh} z-\int d z \operatorname{ch}^{2} z=\operatorname{ch} z \operatorname{sh} z-z-I$, from which it follows

$$
I=\frac{1}{2}(\operatorname{chzsh} z+z)=\frac{1}{2}\left[\sqrt{\frac{|x|}{L}} \sqrt{\frac{|x|}{L}+1}-\operatorname{ash}\left(\sqrt{\frac{|x|}{L}}\right)\right]
$$

So, the function $t(x)$ can be calculated analytically and the function $\mathrm{x}(\mathrm{t})$ (and therefore full $r(t)$ ) can be easily found numerically. However, the result is easily understandable without calculations.

## 3. Result of Investigations



Figure 3. A trajectory of an electron, starting from the point a will be a straight line up to the point $B$, and then it will be curved along Bc if the slit $C D$ is closed. Otherwise the trajectory ac will not change up to the point $b$, but after this point the electron motion will be uniform up to $b$ ' with the speed determined by tangent to the curve ac at the point $b$. Therefore the segment bb' is rectilinear. After $b$ ' the trajectory is again curved, but collision of the electron at the detecting screen will take place at the point d different from $c$, which can be interpreted as interference of slits $A B$ and $C D$.

After these calculations it becomes possible to understand what will be obtained. Some "interference" will be really seen, but it is not interesting. Indeed, let's consider Fig. 3. In our approximation the electron trajectory does not change with opening the second slit, at those parts of trajectory, where the second slit is not directly seen. Therefore the trajectory change in the presence of the second slit is the
same as the trajectory change, when the potential along it is changed. The more interesting will be the case where the electron trajectory changes even when electron flies in the direction opposite to the position, where the second slit is opened. This situation is possible only, when the forces between the electron and the screen contain tangential components, i.e. are not only normal to the screen.

## 4. Discussion

To continue the program formulated in Introduction it is necessary first to solve an electrostatic problem of a point charge in the presence of half infinite ideal conducting plane. It can be also possible to represent the Coulomb field of a point particle in the form of Fourier expansion and find diffraction of every wave component on half planes or straight bands using the Sommerfeld approach [10]. Of course, even in this approach we cannot obtain a diffraction pattern on the observation screen, because we do not have such a parameter as wavelength; however we can get it, if we take into account the retardation of the action on the particle of its own filed, reflected from screen. We can use also the Planck constant, if in numerical calculations we require that the step length $d$, during which particle moves uniformly, is such that the action on it is equal to $h$. Of course, the origin of this constant cannot be found in classical calculations.

## 5. Conclusion

With our first simplest approach to interaction of particle with the screen we did not succeed to get interference on two slits in classical terms, because not all the particle's trajectories are changed by presence of the second slit, however we see a way, how to improve our approximation and invite physics community to join our attempts to resolve this quantum mechanical enigma.

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