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Computation of Normal Depth in a U-Shaped Open Channel Using the Rough Model Method

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Abstract

The rough model method is applied in order to explicitly computing normal depth in a U-Shaped channel. Simple relationships are obtained by using the geometric characteristics of a triangle. The Darcy-Weisbach relationship is applied to a rough model in order to establish the equation of the flow. The resulting equation is implicit towards the aspect ratio and its resolution is possible through a simple numerical procedure which consists in approaching successively the solution. The process of calculation is not constraining since the solution is obtained, in the worst case, at the end of the eighth step of calculation. Once the aspect ratio in the rough model is calculated, the aspect ratio and therefore the non-dimensional normal depth in the current channel is derived through a dimensionless correction factor of linear dimension. A practical example is proposed to better assessing the reliability and the simplicity of the advocated method.

Keywords

Rough Model Method, Normal Depth, U-Shaped Channel, Discharge, Longitudinal Slope, Open Channel

1. Introduction

It is known that the normal depth plays a major role in the classification of varied flow and in the design of canals and conduits. In recent years, the calculation of the normal depth in the channels has been the subject of particular interest by many researchers. This calculation was performed on different channels geometric profiles, such as trapezoidal channel, circular conduit, parabolic channel and many more [1]-[7]. Resulting equations are not easy to handle for the use of the engineer. The computation of normal depth in a U-Shaped channel does no exception to this rule. To solve the problem, algorithms and optimization methods are used leading to complex equations [8].

The U-Shaped channel may be considered as a triangular channel with a rounded bottom. When using the geometric properties of a triangle, one obtains simple relationships governing the hydraulic characteristics of the U-Shaped channel. The form of the equation governing the flow is simple but implied, on which standard numerical or successive approximation methods can then be applied as the fixed-point method. In addition to their complexity, current methods use Manning's relationship, considering the Manning coefficient n as a constant.

This approach is not physically justified because Manning's coefficient depends on geometric and hydraulic flow characteristics such as discharge, slope, kinematic viscosity, normal depth and more particularly the absolute roughness which characterizes the state of the inner wall of the channel. This is the parameter that must be considered in the calculation of the normal depth. It is in this context that this study is proposed, based on a new method known as the Rough Model Method (RMM) which has proven in the recent past in the design of conduits and canals as well as the calculation of normal depth [9]-[21]. All hydraulic flow characteristics in the considered channel are directly derived from those of a referential rough model which are known characteristics. This is made possible through a nondimensional correction factor of linear dimension. In the RMM, there is no restriction in the involved parameters and resulting equations are applicable to the whole domain of the turbulent flow corresponding to Reynolds number $R \ge 2300$ and relative roughness varying in the wide range $0 \le \varepsilon / D_{\mu} \le 0.05$. A calculation example is provided in order to better understand the procedure of computation and to appreciate the simplicity and the reliability of the advocated method.

2. Basic Equations

The relationships on which the study is based are simple well known hydraulic equations namely, Darcy-Weisbach equation [22], Colebrook-White equation [23] and Reynolds number formula. The energy slope of a conduit or channel is given by the Darcy-Weisbach relationship as:

$$i = \frac{f}{D_h} \frac{Q^2}{2gA^2} \tag{1}$$

where Q is the discharge, g is the acceleration due to gravity, A is the wetted area, D_h is the hydraulic diameter and f is the friction factor given by the well known Colebrook-White formula as:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D_h}{3.7} + \frac{2.51}{R\sqrt{f}}\right)$$
(2)

where ε is the absolute roughness and *R* is the Reynolds number which can be expressed as :

$$R = \frac{4Q}{P\nu} \tag{3}$$

where v is the kinematic viscosity and P is the wetted perimeter.

3. Referential Rough Model

All geometric and hydraulic characteristics of the rough model are distinguished by the symbol "". We consider a rough model particularly characterized by $\overline{\epsilon} / \overline{D_h} = 0.037$ as the arbitrarily assigned relative roughness value, where $\overline{D_h}$ is the hydraulic diameter. The chosen relative roughness value is so large that the prevailed flow regime is fully rough. Thus, the friction factor is $\overline{f} = 1/16$ according to Eq. 2 for $R = \overline{R}$ tending to infinitely large value. The rough model is also characterized by the horizontal (delete this word) linear dimension $\overline{r} = r$, a side slope *m* horizontal to 1 vertical and a longitudinal slope $\overline{i} = i$ (Figure 1). The discharge is $\overline{Q} = Q$ implying $\overline{y_n} \neq y_n$ and even $\overline{y_n} > y_n$. The aspect ratio, also known as the non-dimensional normal depth, is thus $\overline{\eta_n} = \overline{y_n} / r \neq \eta_n = y_n / r$.





Figure 1. Normal depth in a U-Shaped Channel. a) Current channel. b) Rough model.

Applying Eq. 1 to the rough model leads to:

$$i = \frac{\overline{f}}{\overline{D_h}} \frac{Q^2}{2g\overline{A}}$$
(4)

Bearing in mind that $\overline{D_h} = 4\overline{A}/\overline{P}$ and $\overline{f} = 1/16$, Eq. 4 can be rewritten as:

$$i = \frac{1}{128g} \frac{\bar{P}}{\bar{A}^3} Q^2$$
(5)

The flow in the rough model is such that $y_n \ge y_1 = r(1 - \cos \theta)$, or else:

$$\frac{1}{y_n} \ge r \left(1 - \frac{m}{\sqrt{1 + m^2}} \right) \tag{6}$$

The vertical linear dimension y_0 (Figure 1) can be expressed as:

$$y_0 = r \left(\frac{\sqrt{1+m^2}}{m} - 1 \right) \tag{7}$$

By geometrical considerations resulting from Figure 1b, one can write the water area \overline{A} in the rough model as:

$$\bar{A} = my^{-2} - m(y_0 + y_1)^2 + r^2(\theta - \sin\theta\cos\theta)$$
(8)

where $\sin \theta = 1/\sqrt{1+m^2}$ and $\cos \theta = m/\sqrt{1+m^2}$. Thus, Eq. 8 is reduced to:

$$\overline{A} = mr^2(\overline{\eta}^2 - \chi_1) \tag{9}$$

Where:

$$\overline{\eta} = \overline{y} / r \tag{10}$$

$$\chi_1 = \frac{1}{m} \left(\frac{1}{m} - \sin^{-1} \frac{1}{\sqrt{1 + m^2}} \right)$$
(11)

From Figure 1b, the wetted perimeter \overline{P} in the rough model can be written as:

$$\overline{P} = 2\overline{y}\sqrt{1+m^2} - 2(y_0 + y_1)\sqrt{1+m^2} + 2r\theta$$
(12)

After rearrangements, Eq. 12 becomes:

$$\overline{P} = 2r\sqrt{1+m^2}(\overline{\eta} - \chi_2)$$
(13)

Where:

$$\chi_2 = \frac{1}{\sqrt{1+m^2}} \left(\frac{1}{m} - \sin^{-1} \frac{1}{\sqrt{1+m^2}} \right)$$
(14)

Inserting Eq. 9 and Eq. 13 into Eq. 5, one may obtain:

$$\frac{(\eta^2 - \chi_1)}{(\eta - \chi_2)^{1/3}} = Q^{*2/3}$$
(15)

 Q^* is the relative conductivity expressed as:

$$Q^* = \frac{(1+m^2)^{1/4}}{8m^{3/2}} \left(\frac{Q}{\sqrt{gir^5}}\right)$$
(16)

All parameters of Eq. 16 are known, which allows calculating the relative conductivity Q^* . What is needed is the determination of the aspect ratio $\overline{\eta}$ using Eq. 15. Let us assume the following change in variables:

$$z = \overline{\eta}^2 - \chi_1 \tag{17}$$

Thus Eq. 15 becomes:

$$z = Q^{*2/3} \left(\sqrt{z + \chi_1} - \chi_2 \right)^{1/3}$$
(18)

Eq. 18 is implicit towards the variable z. To solve Eq. 18 we suggest a numerical method which consists in approaching successively the solution. The calculation process is iterative and operates on Eq. 18 after selecting a first value of z. Assume that the first value of z is z_0 . The calculation showed that the most appropriate value of z_0 is χ_1 . As a result, the next values of z are obtained such that:

$$z_1 = Q^{*2/3} \left(\sqrt{2\chi_1} - \chi_2 \right)^{1/3}$$
$$z_2 = Q^{*2/3} \left(\sqrt{z_1 + \chi_1} - \chi_2 \right)^{1/3} \dots \text{ and so on.}$$

The calculation process stops when z_i and z_{i+1} are sufficiently close. It is obvious that the speed of convergence of the described iterative process depends strongly on the value of z_0 initially selected.

With $z_0 = \chi_1$, intensive calculations showed that almost exact value of z is obtained, in the worst case, at the end of the eighth step of calculating only. The suggested procedure of calculation is not therefore constraining. Once the final value of z is determined, the aspect ratio $\overline{\eta}$ in the rough model is worked out from Eq. 17 as:

$$\overline{\eta} = \sqrt{z + \chi_1} \tag{19}$$

Consequently, the aspect ratio $\overline{\eta_n} = \overline{y_n} / r$ is expressed as:

$$\overline{\eta_n} = \overline{\eta} - y_0 / r \tag{20}$$

Remember that $\overline{\eta}$ is given by Eq. 19 and y_0 / r is governed by Eq. 7. Once $\overline{\eta}$ is determined from Eq. 19, the water area \overline{A} and the wetted perimeter \overline{P} in the rough model are easily computed by the use of Eq. 9 and Eq. 13 respectively, for the given data of *m* and *r*. Consequently, Reynolds number \overline{R} and the hydraulic diameter $\overline{D_h}$ in the rough model can be calculated using the following formulae:

$$\overline{R} = \frac{4Q}{\overline{P}\nu}$$
(21)

$$\overline{D_h} = 4\overline{A} / \overline{P} \tag{22}$$

4. Non-Dimensional Correction Factor of Linear Dimension

The rough model method states that any linear dimension L of a conduit or channel and the linear dimension \overline{L} of its rough model are related by the following equation, applicable to the entire domain of the turbulent flow:

$$L = \psi \overline{L} \tag{23}$$

where ψ is a non-dimensional correction factor of linear dimension, less than unity, which is governed by the following relationship [9, 24]:

$$\psi = 1.35 \left[-\log\left(\frac{\varepsilon / \overline{D_{k}}}{4.75} + \frac{8.5}{\overline{R}}\right) \right]^{-2/5}$$
(24)

All parameters of Eq. 24 are known, allowing to explicitly calculating ψ .

5. Computation Steps of the Normal Depth

To compute the normal depth y_n , the following data must be given: Q, r, i, m, ε and v. Note firstly that these data are easily measurable in practice and secondly the flow resistance coefficient such as Chezy's coefficient or Manning's roughness coefficient is not imposed. To compute the required normal depth y_n , the following steps are recommended:

1. Compute the relative conductivity Q^* by the use of Eq. 16.

2. Compute χ_1 and χ_2 using Eq. 11 and Eq. 14 respectively.

3. Calculate the value of z using Eq. 18 by adopting the described iterative process, considering $z_0 = \chi_1$.

4. With the calculated value of z, compute the aspect ratio $\overline{\eta}$ in the rough model using Eq. 19.

5. As a result, Eq. 9 and Eq. 13 give the water area \overline{A} and the wetted perimeter \overline{P} respectively. This allows deducing Reynolds number \overline{R} and the hydraulic diameter $\overline{D_h} = 4\overline{A}/\overline{P}$ by the use of Eq. 21 and Eq. 22 respectively.

6. Thus, compute the non-dimensional correction factor of linear dimension ψ by applying the explicit Eq. 24.

7. Assign to the rough model the new linear dimension $\bar{r} = r/\psi$ according to Eq. 23 and derive the corresponding value of the relative conductivity Q^* using Eq. 16.

8. With the calculated value of Q^* , compute z according to the step 3.

9. By introducing this value of z in Eq. 19, one obtains the aspect ratio $\overline{\eta}$ in the rough model equal to the aspect ratio η .

10. According to Eq. 20, the aspect ratio η_n of normal water area is as:

 $\eta_n = \eta - y_0 / r$

11. The required normal depth is then $y_n = r\eta_n$.

6. Example

Compute the normal depth y_n in the U-Shaped channel shown in Figure 1a, for the following data:

 $Q = 10 \ m^3 \ / \ s \ , \ r = 0.8 \ m \ , \ i = 5 \times 10^{-4} \ , \ \mathcal{E} = 10^{-3} \ m \ , \ \theta = 45^{\circ} (m = 1) \ ,$ $V = 10^{-6} \ m^2 \ / \ s \ .$

1. According to Eq. 16, the relative conductivity Q^* is:

$$Q^* = \frac{(1+m^2)^{1/4}}{8m^{3/2}} \left(\frac{Q}{\sqrt{gir^5}}\right) = \frac{(1+1^2)^{1/4}}{8\times 1^{3/2}} \left(\frac{10}{\sqrt{9.81\times 5\times 10^{-4}\times 0.8^5}}\right)$$
$$= 37.0785753$$

2. Using Eq. 11 and Eq. 14, the parameters χ_1 and χ_2 are respectively:

$$\chi_1 = \frac{1}{m} \left(\frac{1}{m} - \sin^{-1} \frac{1}{\sqrt{1 + m^2}} \right)$$
$$= \frac{1}{1} \times \left(\frac{1}{1} - \sin^{-1} \frac{1}{\sqrt{1 + 1^2}} \right) = 0.21460184$$
$$\chi_2 = \frac{1}{\sqrt{1 + m^2}} \left(\frac{1}{m} - \sin^{-1} \frac{1}{\sqrt{1 + m^2}} \right)$$
$$= \frac{1}{\sqrt{1 + 1^2}} \times \left(\frac{1}{1} - \sin^{-1} \frac{1}{\sqrt{1 + 1^2}} \right) = 0.15174641$$

3. Inserting the obtained value of Q^* in Eq. 18 and

adopting the described iterative process for $z_0 = \chi_1$, the final value of z is such that: $z_7 \cong z_8 = z = 18.2949812$.

4. According to Eq. 19, the aspect ratio $\overline{\eta}$ in the rough is as:

$$\overline{\eta} = \sqrt{z + \chi_1} = \sqrt{18.2949812 + 0.21460184}$$
$$= 4.30227649$$

5. Using Eq. 9, the water area \overline{A} in the rough model is:

$$\overline{A} = mr^{2}(\overline{\eta}^{2} - \chi_{1})$$
$$= 1 \times 0.8^{2} \times (4.30227649^{2} - 0.21460184) = 11.7087879m^{2}$$

According to Eq. 13, the wetted perimeter \overline{P} is:

$$\overline{P} = 2r\sqrt{1+m^2}(\overline{\eta} - \chi_2)$$
$$= 2 \times 0.8 \times \sqrt{1+1^2} \times (4.30227649 - 0.15174641) = 9.39157748m$$

Thus, the hydraulic diameter $\overline{D_h} = 4\overline{A}/\overline{P}$ is then:

$$\overline{D_h} = 4\overline{A} / \overline{P} = 4 \times 11.7087879 / 9.39157748$$
$$= 4.98693131m$$

Using Eq. 21, Reynolds number \overline{R} is:

$$\overline{R} = \frac{4Q}{\overline{P}_V} = \frac{4 \times 10}{9.39157748 \times 10^{-6}} = 4259135.39$$

6. Eq. 24 gives the non-dimensional correction factor of linear dimension ψ as:

$$\psi = 1.35 \left[-\log\left(\frac{\varepsilon / \overline{D_{k}}}{4.75} + \frac{8.5}{\overline{R}}\right) \right]^{-2/5}$$
$$= 1.35 \times \left[-\log\left(\frac{10^{-3} / 4.98693131}{4.75} + \frac{8.5}{4259135.39}\right) \right]^{-2/5} = 0.74947954$$

7. In accordance with Eq. 23, assign to the rough model the following new value of linear dimension:

$$r = r / \psi = 0.8 / 0.74947954 = 1.06740738m$$

The corresponding value of the relative conductivity Q^* is given by Eq. 16 as:

$$Q^* = \frac{(1+m^2)^{1/4}}{8m^{3/2}} \left(\frac{Q}{\sqrt{gi(r/\psi)^5}} \right)$$
$$= \frac{(1+1^2)^{1/4}}{8\times 1^{3/2}} \left(\frac{10}{\sqrt{9.81\times 5\times 10^{-4}\times 1.06740738^5}} \right)$$
$$= 18.0311115$$

8. Introducing this value of Q^* into Eq. 18 and adopting the described iterative process for $z_0 = \chi_1$, one may obtain at the end of the eighth step of calculation: $z_7 \cong z_8 = z = 9.95865909$.

9. Thus, Eq. 19 gives the aspect ratio η as:

$$\overline{\eta} = \eta = \sqrt{z + \chi_1} = \sqrt{9.95865909 + 0.21460184}$$
$$= 3.18955497$$

10. According to Eq. 20, the aspect ratio η_n of normal water area is as:

$$\eta_n = \eta - y_0 / r = \eta - \left(\frac{\sqrt{1 + m^2}}{m} - 1\right)$$
$$= 3.18955497 - \left(\frac{\sqrt{1 + 1^2}}{1} - 1\right) = 2.77534141$$

11. The required value of normal depth is then:

$$y_n = r\eta_n = 0.8 \times 2.77534141 = 2.22027313m$$

≈ 2.22*m*

 12. This step aims to verify the validity of the calculations by determining the longitudinal slope of the channel using Eq.
 1. The energy slope so calculated should be equal to the slope given in the problem statement.

According to the rough model method, the friction factor f is related to the non-dimensional correction factor ψ by the following formula:

$$f = \psi^5 / 16$$

Hence:

$$f = 0.74947954^5 / 16 = 0.01478015$$

The water area A is given by Eq. 9, for $\eta = \eta = 3.18955497$, as:

$$A = mr^2(\eta^2 - \chi_1)$$

 $= 1 \times 0.8^{2} \times (3.18955497^{2} - 0.21460184) = 6.37354181m^{2}$

According to Eq. 13, the wetted perimeter P is as:

$$P = 2r\sqrt{1+m^2}\left(\eta - \chi_2\right)$$

$$= 2 \times 0.8 \times \sqrt{1 + 1^2} \times (3.18955497 - 0.15174641) = 6.8737761m$$

The hydraulic diameter $D_h = 4A / P$ is thus:

$$D_h = 4A / P = 4 \times 6.37354191 / 6.8737761$$

Finally, according to Eq. 1, the longitudinal slope *i* is:

$$F = \frac{f}{D_h} \frac{Q^2}{2gA^2}$$
$$= \frac{0.01478015}{3.70890278} \times \frac{10^2}{2 \times 9.81 \times 6.37354181^2} = 0.0005$$

As one can observe, the longitudinal slope so calculated is equal to the slope given in the problem statement, confirming the validity of the calculations.

7. Conclusion

The rough model method (RMM) has been successfully applied to compute normal depth in a U-Shaped channel. The calculation was carried with a minimum of practical data, taking into account the effect of the absolute roughness. The resistance coefficients such that Chezy's coefficient or Manning's roughness coefficient were not necessary. The application of the rough model method was based on simple equations of hydraulic which are Darcy-Weisbach relationship, Colebrook-White equation and Reynolds number formula. The Darcy-Weisbach relationship was applied to a referential rough model whose hydraulic and geometric characteristics are known. This led to an implicit equation relating the aspect ratio to the relative conductivity. This equation was solved by a short iterative process based on an initial value carefully chosen. From the aspect ratio of the rough model, the nondimensional normal depth and therefore normal depth in the current channel has been immediately deducted. The calculation procedure was clearly explained through a practical example which further demonstrated the reliability and the simplicity of the rough model method.

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