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Closer Look at EPR Paradox and Bell's Inequality

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Abstract

The EPR paradox is shown to be a result of the wrong definitions in quantum mechanics of such physical quantities as momentum and position. The corrected definitions show that momentum and position of particles can be precisely defined simultaneously and uncertainty relations do not prevent it. Therefore, with correct definition of physical quantities in quantum mechanics, the paradox disappears. The Bohm Aharonov version of the EPR paradox is illustrated by the example of measuring the polarization of photon pairs. The common view of the two photon state radiated by a source is presented. The common Bell's inequality is discussed and the simplest version of it for a specific hidden variable is derived. A possibility of an experimental study of whether a photon and its polarization are preexistent before measurements is considered.

Keywords

Quantum Mechanics, Uncertainty Relations, EPR Paradox, Bell's Inequality, Hidden Parameters, Photon Polarization

1. Introduction

A lot of discussions are going in literature about uncertainty relations, EPR paradox [1] and Bell's inequalities [2]. They are related to a mystical attitude toward quantum mechanics (QM), which seems is supported by experiments [3,4]. I show here some misconception, which prevails all these discussions, propose the simplest Bell's inequality, which can be checked in a single measurement, and describe an experiment to elucidate the situation. The next section presents a critical examination [5-7] of the first EPR paper [1].

2. Criticism of EPR

The paradox was formulated in the paper [1] as follows: It is possible to predict existence of particles with simultaneously precisely defined momentum and position, but, according to uncertainty relations, momentum and position cannot be simultaneously precisely defined, therefore Quantum mechanics is not complete. It must be completed by adding some not yet known, i.e. hidden, parameters.

I show here that particles can really have simultaneously precisely defined momentum and position, and uncertainty relations have nothing to do with it. Therefore paradox does not exist.

If ψ is an eigenfunction of the operator A, that is, if

$$\psi' \equiv A\psi = a\psi, \qquad (1)$$

where a is a number, then the physical quantity A has with certainty the value a whenever the particle is in the state given by ψ.

Figure 1. *Citation from EPR paper [1].*

Let, for example,

$$\psi = e^{(2\pi i/h)p_0 x}, \qquad (2)$$

where h is Planck's constant, p_0 is some constant number, and x the independent variable. Since the operator corresponding to the momentum of the particle is

$$p = (h/2\pi i)\partial/\partial x, \qquad (3)$$

we obtain

$$\psi' = p\psi = (h/2\pi i)\partial\psi/\partial x = p_0\psi. \qquad (4)$$

Thus, in the state given by Eq. (2), the momentum has certainly the value p_0. It thus has meaning to say that the momentum of the particle in the state given by Eq. (2) is real.

Figure 2. *Citation from EPR paper [1].*

The EPR paradox stems from a wrong definition of particle's momentum and position, which leads to an error. The error is clearly visible in [1], but according to some plot, it is not noticed by scientific community.

EPR define a physical quantity represented by an operator A as an eigen value of this operator, which means that it exists only in the states, which are described by eigenfunctions of the operator A (fig.1) independently, whether one deals with compact, like spin, or noncompact spaces, like momentum. With such a definition momentum of a particle exists only in states, described by plane waves as shown in fig.2.

However in such states it is impossible to define a position of the particle. EPR define a "relative probability" to find the particle in an interval [a,b] as shown in fig.3.

In accordance with quantum mechanics we can only say that the relative probability that a measurement of the coordinate will give a result lying between a and b is

$$P(a,b)=\int_a^b \bar{\psi}\psi dx=\int_a^b dx=b-a. \qquad (6)$$

Since this probability is independent of a, but depends only upon the difference $b-a$, we see that all values of the coordinate are equally probable.

Figure 3. *Citation from EPR paper [1].*

This definition is an error, which is not noticed by the whole science community. The value (6) is dimensional and not normalizable. The textbooks teach students, that plane wave state must be modified to $\psi(x)=\exp(ikx)/\sqrt{L}$, where L is some large scale. In this case Eq. (6) of fig.3 looks like (b − a)/L. It is dimensionless, but to be normalizable the distance b-a can not be larger than L. It means that the space is limited by impenetrable walls, but between such walls the wave function is $\psi(x) = \sin(\pi nx/L)$, where n is an integer. Such a function is not an eigenfunction of the momentum operator. Therefore, according to the EPR definition, the particles in the limited space cannot have a momentum.

It is common to read in textbooks that Quantum mechanics deals with Hilbert space, i.e. with normalizable wave functions, called wave packets

$$\int_{-\infty}^{+\infty} |\psi(x)|^2 dx = 1, \qquad (1)$$

therefore

$$\int_a^b |\psi(x)|^2 dx \neq b-a. \qquad (2)$$

No wave packet is an eigenfunction of the momentum operator and therefore, according to EPR definition, particles in Hilbert space cannot have a momentum. Such a Quantum mechanics is not able to describe physical reality. The break of the deadlock is trivial. We must abandon EPR definition and define the values of momentum and position according to J. von Neumann prescription [8] as averages over state wave function

$$p=\int \psi^+(x)\hat{p}\psi(x)dx. \qquad (3)$$

With such a definition particles can have precise position and momentum simultaneously. For example, the Gaussian wave packet has precisely defined velocity, which is equal to classical speed: the space shift of the packet per unit time. It also has the precisely defined position, which can be defined as the position of the packet maximum. The well known uncertainty relation $\Delta p\Delta x \geq h/4$ has no physical meaning. It is only a mathematical theorem, which relates the width of a wave packet to that of its Fourier image.

Of course, with definition (3) we have dispersion

$$\Delta p=\sqrt{\int \psi^+(x)\left(\hat{p}^2-p^2\right)\psi(x)dx}. \qquad (4)$$

But it is not a statistical dispersion. It is a property of the wave packet. Let's consider a classical extended object. Its position is a matter of definition, and dispersion of the position is a characteristic of the object size and shape. Only in a set of similar objects, the dispersion acquires a statistical part.

3. EPR Paradox with Photon Pairs

Now let's look at the Bohm Aharonov version [9] of EPR paradox with photons. Consider a photon pair, which is emitted by one of atoms located at the point z = 0. Photons travel in opposite directions along the z-axis. One photon goes to the point z = A to the experimenter Alice, and the other one goes to the point z =-B to the experimenter Bob as is shown in fig.4. Photons have the same direction of the linear polarization uniformly distributed over the azimuthal angle in the plane (x,y), perpendicular to the direction of the photons propagation.

Alice and Bob are using birefringent calcite crystals to measure polarization of the photons arriving to them. A calcite crystal has two mutually perpendicular axes: ordinary one (denoted by direction of the unit vector o) and extraordinary one (denoted by n). Assume that both axes lie in the plane (x, y). If a coming photon is polarized along o axis, it is registered by detector D_1, and if the coming photon is polarized along n, it is registered by the detector D_2. Axes of the crystals can be rotated around the z-axis.

The EPR paradox can be illustrated as follows. Imagine that the two calcites of Alice and Bob are oriented parallel to each other, and Alice is somewhat closer to the source of the photons than Bob, i.e. she is the first who measures photon of the pair. Imagine two photons flying to A and B are polarized along a vector γ, which is not parallel to either o_A or n_A of Alice's calcite, i.e. $\gamma = \alpha o_A + \beta n_A$, where α and β are the coordinates of the vector γ in the basis of two orthogonal unit vectors. With probability $|\alpha|^2$ the photon at Alice will be registered by the detector D_{1A}, and with probability $|\beta|^2$ by the detector D_{2A}, but it will be counted only by a single detector. Let's say it will be registered by the detector D_{1A}. Then, according to the nonlocal quantum mechanics, since the two photon must have the same polarization, the photon flying to Bob instantly becomes polarized along o_A, and as Bob's calcite is oriented parallel to that of Alice, the photon will be with certainty registered only by D_{1B}, though without Alice's measurement the photon at Bob could with probability $|\beta|^2$ be registered by the detector D_{2B}.

It means that measurement of Alice instantly changes photon polarization at Bob, no matter how great is the distance $A + B$ therebetween. This mental phenomenon, called nonlocality, is the essence of the EPR paradox. From this it follows that, either the theory of relativity is not true, because there are signals propagating with arbitrarily high speed, or quantum mechanics is not complete, i.e. it is necessary to provide to it additional yet unknown, i.e. hidden parameters.

Though, speaking about a photon and its polarization vector γ, we had already introduced the parameter: the azimuthal angle of the vector in the plane (x,y), it did not help us.

In fact, in quantum mechanics a source is supposed to emit not photons. It emits only a two photon entangled wave function

$$|\psi(1,2)\rangle = |\mathbf{x}\rangle_1 |\mathbf{x}\rangle_2 + |\mathbf{y}\rangle_1 |\mathbf{y}\rangle_2 , \quad (5)$$

where x, y are two arbitrary orthogonal unit vector in the (x,y) plane. This function is normalized to 2, because the number of photons is 2. Such a wave function means that the photons with the same amplitude probability can be polarized along any of two mutually perpendicular vectors x and y. Only after measurement by Alice the photon flying to Bob acquires an individual wave function. Suppose that Alice's photon is registered, for example, by the detector D_{1A}, then the wave function of a particle moving to Bob, becomes

$$\begin{aligned} |\psi(2)\rangle &= \langle \mathbf{o}_A || \psi(1,2)\rangle = \langle \mathbf{o}_A | \left(|\mathbf{x}\rangle_1 |\mathbf{x}\rangle_2 + |\mathbf{y}\rangle_1 |\mathbf{y}\rangle_2 \right) \\ &= \alpha_A |\mathbf{x}\rangle_2 + \beta_A |\mathbf{y}\rangle_2 \end{aligned} \quad (6)$$

where

$$\alpha_A = \langle \mathbf{o}_A || \mathbf{x}\rangle_{12} \qquad \beta_A = \langle \mathbf{o}_A || \mathbf{y}\rangle_1 , \quad (7)$$

i.e. it takes the form of a photon with a definite polarization along the unit vector $\boldsymbol{\gamma}_B = \alpha_A \mathbf{x} + \beta_A \mathbf{y}$, which is parallel to the orientation of the axis o_A of the Alice's calcite. This photon wave function (6) is normalized to unity. If, as is usual [10,11], the wave function (5) is normalized to unity (in this case, it should contain the factor $1/\sqrt{2}$), then the wave function (6) is normalized to 1/2, which is unacceptable. However, according to Copenhagen interpretation of quantum mechanics [12], even the object flying to Bob becomes a photon only after measurement by Bob, i.e. when it ceases to exist. We neglect this opinion, and consider the photon and its polarization to preexist before any measurement. So we see that the measurement by Alice instantly predefines Bob's photon polarization, regardless of how far it is, i.e. the effect of the measurement propagates with an arbitrarily high speed. However, the contradiction with the theory of relativity like does not occur because using such measurements it is impossible to transmit some information.

Indeed, let's imagine that Bob and Alice oriented in parallel the axes of their crystals. If Bob registers a photon by the detector D_{1B}, he can be sure that Alice has registered a photon by the detector D_{1A}, but this fact does not depend on the will of Alice, and therefore no information she can send to Bob.

To test the validity of the above, Alice and Bob can make measurements with many emitted photon pairs and compare whether each pair of photons is detected with the same detectors. Of course it is necessary to mark each emitted photon pair, which can be done, for example, by fixing the registration time of photons.

However, experiments on the EPR paradox are not carried out in such a way, but by measuring some inequality to verify as to whether the measurement result contradicts the prediction by Bell [2].

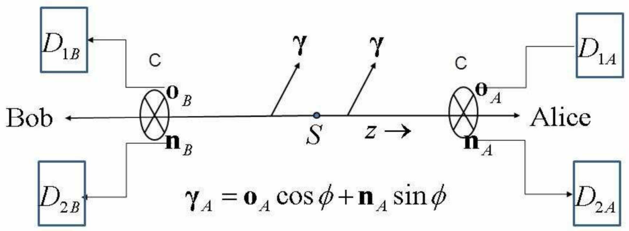

Figure 4. *The scheme of an experiment to demonstrate the EPR paradox in nonlocal quantum mechanics. S - the source of photon pairs with parallel polarization γ, flying in two opposite directions to the experimenters Alice and Bob. C - birefringent calcite crystals, analyzing polarization. The crystals have an ordinary, o, and extraordinary, n, axes. Photons polarized along o, are registered by the detectors D_1, and photons polarized along n, are recorded by detectors D_2. Alice is closer to the source and registers her photon polarization γ_A the first. φ is the azimuthal angle of the polarization vector.*

4. Bell's Inequality

We assign the number +1 to a photon, if after an analyzer it is registered by detector D_1, and-1, if after the analyzer it is registered by detector D_2. Imagine that Alice and Bob conducted 4 experiments: Alice oriented the ordinary axis of her calcite in the direction of two unit vectors a and a*'*, and Bob for each of these orientations of Alice's calcite orienited ordinary axis of his calcite along directions b and b*'*. Bell's inequality appears as follows [10,13]: to photons incident on the crystal with the axis a it is assigned the stohastic value $a = \pm 1$, and similarly for other axes. Then, denoting

$$M = a(b+b') + a'(b-b')\,, \tag{8}$$

where all the letters on the right side correspond to one of the values ±1, one can write the following obvious equality [10,13]

$$|M| = 2\,. \tag{9}$$

After averaging of the expression (9) over many photons, one gets inequality

$$-2 \le \langle ab\rangle + \langle ab'\rangle + \langle a'b\rangle - \langle a'b'\rangle \le 2\,, \tag{10}$$

where the sign <> denotes averaging.

According to nonlocal quantum mechanics, in the case, when the angle between axes a and b is equal to θ_{ab}, if a photon at Alice side is detected, for example, by the detectorD_{1A}, then at Bob's side his photon with probability $\cos^2\theta_{ab}$ is counted by the detector D_{1B} and with probability $\sin^2\theta_{ab}$ by the detector D_{2B}, i.e. (for registration by any detector at Alice side) Bob measures the correlation function

$$E(a,b) \equiv \langle ab\rangle = \cos^2\theta_{ab} - \sin^2\theta_{ab} = \cos(2\theta_{ab})\,. \tag{11}$$

Selecting the azimuthal angles of crystals as shown in fig.5: $\theta_a = \pi/8$, $\theta_b = 0$, $\theta_{a'} = -\pi/8$, $\theta_{b'} = \pi/4$, one obtains

$$\begin{aligned}&\langle ab\rangle + \langle ab'\rangle + \langle a'b\rangle - \langle a'b'\rangle = \\ &\cos^2\theta_{ab} + \cos^2\theta_{ab} + \cos^2\theta_{ab} - \cos^2\theta_{ab} = \frac{4}{\sqrt{2}} > 2,\end{aligned} \tag{12}$$

which contradicts the inequality (10). Many published experiments [3,4] evidence in favor of (12), i.e. in favor of nonlocality in quantum mechanics, and infinite speed of propagation of the influence of manipulation by Alice with her analyzer, which nevertheless does not contradict the finite speed of propagation of the signal in theory of relativity.

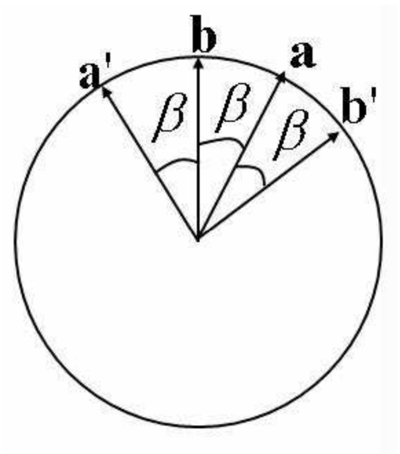

Figure 5. *Directions of ordinary axes in Alice and Bob 4 experiments.*

However, the inequality (10) is too complicated. It is possible to simplify it so that it can be checked in a single measurement.

5. Bell's Inequality for a Particular Hidden Variable

For some particular hidden variable in local Quantum mechanics it is possible to derive the inequality

$$|\langle ab\rangle| \le 1/2. \tag{13}$$

It immediately follows from (13) that (10) is also valid, but (13) is sufficient.

To prove the inequality (13) imagine that the source emits, as it was tacitly expected at the beginning of this article, the pairs of real photons instead of combination (5). In this case, the wave function of photons can be represented by the product of the two individual photons

$$|\psi(1,2)\rangle = |\gamma_1\gamma_2\rangle, \tag{14}$$

With the same polarization which has a uniform probability distribution in the azimuthal angle φ ("hidden" parameter) in the plane (x,y). For a given φ the geometry is shown in fig.6.

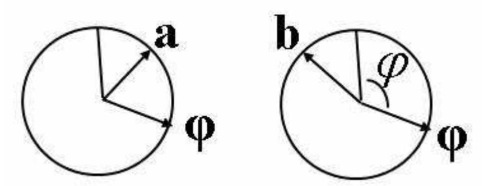

Figure 6. *The geometry of orientation of two ordinary axes of Alice's and Bob's calcite and of photon pair polarization.*

In this case, the photon polarization $\gamma_1(\varphi)$, registered by Alice, according to the local quantum mechanics, is characterized by the correlation function

$$E(a,\phi) = \cos^2(\phi - \theta_a) - \sin^2(\phi - \theta_a) = \cos 2(\phi - \theta_a), \tag{15}$$

and photons detected by Bob, according to the local quantum mechanics, is characterized by the correlation function

$$E(b,\phi) = \cos^2(\phi - \theta_b) - \sin^2(\phi - \theta_b) = \cos 2(\phi - \theta_b). \tag{16}$$

The correlation function of both photons counted by Alice and Bob is

$$E(a,b) = \langle ab\rangle = \int_0^{2\pi} \frac{d\phi}{2\pi} \cos 2(\phi - \theta_a)\cos 2(\phi - \theta_b) = \frac{1}{2}\cos 2(\theta_{ab}) \tag{17}$$

This expression satisfies the inequality (13), and thus satisfies the inequality (10).

The resulting expression differs from (11) only by a constant factor. If it can be extracted, then the violation of the Bell's inequality can be checked in a single measurement at $\theta_{ab} < \pi/6$.

If extraction of this factor in experiments is impossible, then the check of the inequality (10) is also impossible. It is nevertheless possible to distinguish the local quantum mechanics with individual photons (14) and the "hidden" parameter φ from the nonlocal quantum mechanics without this parameter and with the wave function (5).

In the case of independent photons and parallel aligned crystals counting of a photon by the detector D_{1A} at Alice side does not prohibit registration of the photon at Bob side by the detector D_{2B}. The probability of such registration is

$$P(a_1,b_2)=\int_0^{2\pi}\frac{d\phi}{2\pi}\cos 2(\phi-\theta_a)\cos 2(\phi-\theta_b)=\frac{1}{4}\left(1-\frac{1}{2}\cos 2(\theta_{ab})\right)=\frac{1}{8}, \quad (18)$$

while the probability of registration by the same detectors is equal to

$$P(a_1,b_1)=\int_0^{2\pi}\frac{d\phi}{2\pi}\cos 2(\phi-\theta_a)\cos 2(\phi-\theta_b)=\frac{1}{4}\left(1+\frac{1}{2}\cos 2(\theta_{ab})\right)=\frac{3}{8}, \quad (19)$$

So for independent photons the ratio of the probability of registration by different detectors to registration by the same detectors is 1/3.

6. Conclusion

From the above it follows that for a final check of two versions of quantum mechanics with individual and entangled photons it is required an experiment with parallel aligned crystals and rare pulses of the photon pairs, where each pair is marked by the time of registration, and Alice takes measurements the first and accepts only photons registered, say, by the detectorD_{1A}. Only these photon pairs are accepted by Bob, whose task is to check whether some of these photons are registered by his detector D_{2B}. In this experiment, there will be no loop holes, and one of the types of quantum mechanics will be completely ruled out.

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