

# To the non-local theory of levitation

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## Abstract

In this paper we revisit the levitation phenomenon using the generalized Boltzmann kinetics theory which can represent the non-local physics of this levitation phenomenon. This approach can identify the conditions when the levitation can take place under the influence of correlated electromagnetic and gravitational fields. The sufficient mathematical conditions of levitation are obtained. It means that the regime of levitation could be realized from the position of the non-local hydrodynamics.

## Keywords

Foundations of the Theory of Transport Processes, Generalized Hydrodynamic Equations, Basements of Non-Local Physics, Levitation

## 1. Introduction

The phenomenon of levitation has attracted attention from philosophers and scientists in the past and now. How can levitation be possible? What power or agent accomplishes it? The most obvious explanation - the possession of a word of mystical power - is not interesting here for us.

Levitation is the process by which an object is suspended by a physical force against gravity, in a stable position without solid physical contact. Many periodicals related to the levitation problems exist, for example IEEE Transactions on: Applied Superconductivity, Automation Science and Engineering, Control Systems Technology, Magnetics.

In spite of the tremendous recent advances, notably in power electronics, magnetic materials, on the application of electromagnetic suspension and levitation techniques to advanced ground transportation, physics of levitations needs in following significant investigations. In this paper we revisit the levitation phenomenon using the generalized Boltzmann kinetics theory [1 – 4] which can represent the non-local physics of this levitation phenomenon.

The investigations of the levitation stability have a long history and are considered in details in [5 – 9]. As usual the problem review begins with the citation of the Earnshaw paper [5]. Earnshaw's theorem depends on a mathematical

property of the  $1/r$  type energy potential valid for magnetostatic and electrostatic events and gravitation. At any point where there is force balance is equal to zero, the equilibrium is unstable because there can be no local minimum in the potential energy. There must be some loopholes though, because magnets above superconductors and the magnet configuration do stably levitate including frogs [6] and toys like levitron (spinning magnet tops), flying globe and so on [10, 11]. It means that diamagnetic material can stabilize the levitation of permanent magnets. It is well known that the potential energy density of the magnetic field can be written as

$$w_m = -\mathbf{M} \cdot \mathbf{B} \quad (1.1)$$

where  $\mathbf{B}$  is magnetic induction,  $\mathbf{M}$  is magnetization. Using the phenomenological relation

$$\mathbf{M} = \chi \mathbf{H}, \quad (1.2)$$

where  $\chi$  is magnetic susceptibility, we have for the unit volume of a magnetic material

$$w_m = -\frac{\chi}{\mu\mu_0} B^2 \quad (1.3)$$

The force acting on the unit volume of a levitating object is

$$\mathbf{F} = \frac{\chi}{\mu_0\mu} \text{grad} B^2, \quad (1.4)$$

if the phenomenological parameters are constant. Diamagnets (for which  $\chi < 0$ ) are repelled by magnetic fields and attracted to field minima. As a result, diamagnets can satisfy the stability conditions [6 – 9] and the following conditions are exceptions to Earnshaw's theorem:

a) Diamagnetism occurs in materials which have a relative permeability less than one. The result is that eddy currents are induced in a diamagnetic material, it will repel magnetic flux.

b) The Meissner effect which occurs in superconductors. Superconductors have zero internal resistance. As such induced currents tend to persist, and as a result the magnetic field they cause will persist as well.

c) As result of oscillations, when an alternating current is passed through an electromagnet, it behaves like a diamagnetic material.

d) Rotation: employed by the Levitron, it uses gyroscopic motion to overcome levitation instability.

e) Feedback can be used in conjunction with electromagnets to dynamically adjust magnetic flux in order to maintain levitation.

The main shortcoming of the Earnshaw theory consists in application of principles of local physics to the non-equilibrium non-local statistical systems.

The aim of this paper consists in application of the non-local physics methods to the effect of levitation. We intend to answer two questions:

1) Is it possible to formulate the sufficient conditions of levitation from the position of the unified non-local theory of transport processes (UNTT) [see, for example, 1 – 4]?

2) Is it possible to speak about the mutual influence of electromagnetic field and gravitation in the frame of UNTT?

## 2. Basic Equations

Non-local hydrodynamic equations have the form [1 – 4]:  
(continuity equation for a mixture)

$$\begin{aligned} & \frac{\partial}{\partial t} \left\{ \rho - \sum_{\alpha} \tau_{\alpha}^{(0)} \left[ \frac{\partial \rho_{\alpha}}{\partial t} + \frac{\partial}{\partial \mathbf{r}} \cdot (\rho_{\alpha} \mathbf{v}_0) \right] \right\} \\ & + \frac{\partial}{\partial \mathbf{r}} \cdot \left\{ \rho \mathbf{v}_0 - \sum_{\alpha} \tau_{\alpha}^{(0)} \left[ \frac{\partial}{\partial t} (\rho_{\alpha} \mathbf{v}_0) + \frac{\partial}{\partial \mathbf{r}} \cdot (\rho_{\alpha} \mathbf{v}_0 \mathbf{v}_0) + \tilde{\mathbf{I}} \cdot \frac{\partial \rho_{\alpha}}{\partial \mathbf{r}} \right. \right. \\ & \left. \left. - \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} - \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha} \mathbf{v}_0 \times \mathbf{B} \right] \right\} = 0, \end{aligned} \quad (2.1)$$

(motion equation)

$$\begin{aligned} & \frac{\partial}{\partial t} \left\{ \rho \mathbf{v}_0 - \sum_{\alpha} \tau_{\alpha}^{(0)} \left[ \frac{\partial}{\partial t} (\rho_{\alpha} \mathbf{v}_0) + \frac{\partial}{\partial \mathbf{r}} \cdot (\rho_{\alpha} \mathbf{v}_0 \mathbf{v}_0) + \frac{\partial \rho_{\alpha}}{\partial \mathbf{r}} - \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} \right. \right. \\ & \left. \left. - \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha} \mathbf{v}_0 \times \mathbf{B} \right] \right\} - \sum_{\alpha} \mathbf{F}_{\alpha}^{(1)} \left[ \rho_{\alpha} - \tau_{\alpha}^{(0)} \left( \frac{\partial \rho_{\alpha}}{\partial t} + \frac{\partial}{\partial \mathbf{r}} \cdot (\rho_{\alpha} \mathbf{v}_0) \right) \right] \\ & - \sum_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} \left\{ \rho_{\alpha} \mathbf{v}_0 - \tau_{\alpha}^{(0)} \left[ \frac{\partial}{\partial t} (\rho_{\alpha} \mathbf{v}_0) + \frac{\partial}{\partial \mathbf{r}} \cdot (\rho_{\alpha} \mathbf{v}_0 \mathbf{v}_0) + \frac{\partial \rho_{\alpha}}{\partial \mathbf{r}} - \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} \right. \right. \\ & \left. \left. - \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha} \mathbf{v}_0 \times \mathbf{B} \right] \right\} \times \mathbf{B} + \frac{\partial}{\partial \mathbf{r}} \cdot \left\{ \rho \mathbf{v}_0 \mathbf{v}_0 + p \tilde{\mathbf{I}} - \sum_{\alpha} \tau_{\alpha}^{(0)} \left[ \frac{\partial}{\partial t} (\rho_{\alpha} \mathbf{v}_0 \mathbf{v}_0 \right. \right. \\ & \left. \left. + p_{\alpha} \tilde{\mathbf{I}}) + \frac{\partial}{\partial \mathbf{r}} \cdot (\rho_{\alpha} \mathbf{v}_0 \mathbf{v}_0) + 2 \tilde{\mathbf{I}} \cdot \left( \frac{\partial}{\partial \mathbf{r}} \cdot (\rho_{\alpha} \mathbf{v}_0) \right) + \frac{\partial}{\partial \mathbf{r}} \cdot (\tilde{\mathbf{I}} p_{\alpha} \mathbf{v}_0) \right. \right. \\ & \left. \left. - \mathbf{F}_{\alpha}^{(1)} \rho_{\alpha} \mathbf{v}_0 - \rho_{\alpha} \mathbf{v}_0 \mathbf{F}_{\alpha}^{(1)} - \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha} [\mathbf{v}_0 \times \mathbf{B}] \mathbf{v}_0 - \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha} \mathbf{v}_0 [\mathbf{v}_0 \times \mathbf{B}] \right] \right\} = 0 \end{aligned} \quad (2.2)$$

(energy equation)

$$\begin{aligned} & \frac{\partial}{\partial t} \left\{ \frac{\rho v_0^2}{2} + \frac{3}{2} p + \sum_{\alpha} \varepsilon_{\alpha} n_{\alpha} - \sum_{\alpha} \tau_{\alpha}^{(0)} \left[ \frac{\partial}{\partial t} \left( \frac{\rho_{\alpha} v_0^2}{2} + \frac{3}{2} p_{\alpha} + \varepsilon_{\alpha} n_{\alpha} \right) \right. \right. \\ & \left. \left. + \frac{\partial}{\partial \mathbf{r}} \cdot \left( \frac{1}{2} \rho_{\alpha} v_0^2 \mathbf{v}_0 + \frac{5}{2} p_{\alpha} \mathbf{v}_0 + \varepsilon_{\alpha} n_{\alpha} \mathbf{v}_0 \right) - \mathbf{F}_{\alpha}^{(1)} \cdot \rho_{\alpha} \mathbf{v}_0 \right] \right\} \\ & + \frac{\partial}{\partial \mathbf{r}} \cdot \left\{ \frac{1}{2} \rho v_0^2 \mathbf{v}_0 + \frac{5}{2} p \mathbf{v}_0 + \mathbf{v}_0 \sum_{\alpha} \varepsilon_{\alpha} n_{\alpha} - \sum_{\alpha} \tau_{\alpha}^{(0)} \left[ \frac{\partial}{\partial t} \left( \frac{1}{2} \rho_{\alpha} v_0^2 \mathbf{v}_0 \right. \right. \right. \\ & \left. \left. + \frac{5}{2} p_{\alpha} \mathbf{v}_0 + \varepsilon_{\alpha} n_{\alpha} \mathbf{v}_0 \right) + \frac{\partial}{\partial \mathbf{r}} \cdot \left( \frac{1}{2} \rho_{\alpha} v_0^2 \mathbf{v}_0 \mathbf{v}_0 + \frac{7}{2} p_{\alpha} \mathbf{v}_0 \mathbf{v}_0 + \frac{1}{2} p_{\alpha} v_0^2 \tilde{\mathbf{I}} \right. \right. \\ & \left. \left. + \frac{5}{2} \frac{p_{\alpha}}{\rho_{\alpha}} \tilde{\mathbf{I}} + \varepsilon_{\alpha} n_{\alpha} \mathbf{v}_0 \mathbf{v}_0 + \varepsilon_{\alpha} \frac{p_{\alpha}}{m_{\alpha}} \tilde{\mathbf{I}} \right) - \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} \cdot \mathbf{v}_0 \mathbf{v}_0 - p_{\alpha} \mathbf{F}_{\alpha}^{(1)} \cdot \tilde{\mathbf{I}} \right. \\ & \left. - \frac{1}{2} \rho_{\alpha} v_0^2 \mathbf{F}_{\alpha}^{(1)} - \frac{3}{2} \mathbf{F}_{\alpha}^{(1)} p_{\alpha} - \frac{\rho_{\alpha} v_0^2}{2} \frac{q_{\alpha}}{m_{\alpha}} [\mathbf{v}_0 \times \mathbf{B}] - \frac{5}{2} p_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} [\mathbf{v}_0 \times \mathbf{B}] \right. \\ & \left. - \varepsilon_{\alpha} n_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} [\mathbf{v}_0 \times \mathbf{B}] - \varepsilon_{\alpha} n_{\alpha} \mathbf{F}_{\alpha}^{(1)} \right] \right\} - \mathbf{v}_0 \cdot \sum_{\alpha} \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} \\ & + \sum_{\alpha} \tau_{\alpha}^{(0)} \mathbf{F}_{\alpha}^{(1)} \cdot \left[ \frac{\partial}{\partial t} (\rho_{\alpha} \mathbf{v}_0) + \frac{\partial}{\partial \mathbf{r}} \cdot (\rho_{\alpha} \mathbf{v}_0 \mathbf{v}_0) + \frac{\partial}{\partial \mathbf{r}} \cdot p_{\alpha} \tilde{\mathbf{I}} - \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} - q_{\alpha} n_{\alpha} [\mathbf{v}_0 \times \mathbf{B}] \right] = 0. \end{aligned} \quad (2.3)$$

where  $\mathbf{v}_0$  is the hydrodynamic velocity in the coordinate system at rest,  $\rho_{\alpha}$  is the density of  $\alpha$  - species,  $p$  is the pressure,  $\tilde{\mathbf{I}}$  - unit tensore,  $\mathbf{F}_{\alpha}^{(1)}$  is the force of the non-nonmagnetic origin acting on the unit of volume,  $\varepsilon_{\alpha}$  is the internal energy of a particle of the  $\alpha$  - species,  $\tau$  is non-local parameter.

Important remarks:

1. Equations (2.1) – (2.3) should be considered as local approximation of non-local equations (NLE) written in the hydrodynamic form. NLE include quantum hydrodynamics of Schrödinger – Madelung as a deep particular case [4] and can be applied in the frame of the unified theory from the atom scale to the Universe evolution.

2. The basic system contains the cross terms for the forces of the mass and electro- magneto-dynamic origin. It means that the fluctuation of the gravitational field leads to the electro- magneto dynamical fluctuations and verse versa.

Sufficient conditions of levitation can be obtained from Eqs. (2.1) – (2.3) after equalizing all terms containing forces to zero. Namely, from the continuity equation

$$\frac{\partial}{\partial \mathbf{r}} \cdot \left\{ \sum_{\alpha} \tau_{\alpha}^{(0)} \left[ \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} + \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha} \mathbf{v}_0 \times \mathbf{B} \right] \right\} = 0, \quad (2.4)$$

from the motion equation follows

$$\begin{aligned} & \frac{\partial}{\partial t} \left\{ \sum_{\alpha} \tau_{\alpha}^{(0)} \left[ \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} + \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha} \mathbf{v}_0 \times \mathbf{B} \right] \right\} - \sum_{\alpha} \mathbf{F}_{\alpha}^{(1)} \left[ \rho_{\alpha} - \tau_{\alpha}^{(0)} \left( \frac{\partial \rho_{\alpha}}{\partial t} + \frac{\partial}{\partial \mathbf{r}} \cdot (\rho_{\alpha} \mathbf{v}_0) \right) \right] \\ & - \sum_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} \left\{ \tau_{\alpha}^{(0)} \left[ \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} + \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha} \mathbf{v}_0 \times \mathbf{B} \right] \right\} \times \mathbf{B} \\ & + \frac{\partial}{\partial t} \left\{ \sum_{\alpha} \tau_{\alpha}^{(0)} \left[ \mathbf{F}_{\alpha}^{(1)} \rho_{\alpha} \mathbf{v}_0 + \rho_{\alpha} \mathbf{v}_0 \mathbf{F}_{\alpha}^{(1)} + \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha} [\mathbf{v}_0 \times \mathbf{B}] \mathbf{v}_0 + \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha} \mathbf{v}_0 [\mathbf{v}_0 \times \mathbf{B}] \right] \right\} = 0, \end{aligned} \quad (2.5)$$

and from the energy equation we find

$$\begin{aligned} & \frac{\partial}{\partial t} \sum_{\alpha} \tau_{\alpha}^{(0)} (\rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} \cdot \mathbf{v}_0) \\ & + \frac{\partial}{\partial t} \cdot \left\{ \sum_{\alpha} \tau_{\alpha}^{(0)} \left[ \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} \cdot \mathbf{v}_0 \mathbf{v}_0 + \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} \cdot \mathbf{I} + \frac{1}{2} \rho_{\alpha} v_0^2 \mathbf{F}_{\alpha}^{(1)} + \frac{3}{2} \mathbf{F}_{\alpha}^{(1)} p_{\alpha} + \frac{\rho_{\alpha} v_0^2}{2} \frac{q_{\alpha}}{m_{\alpha}} [\mathbf{v}_0 \times \mathbf{B}] \right. \right. \\ & \left. \left. + \frac{5}{2} p_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} [\mathbf{v}_0 \times \mathbf{B}] + \varepsilon_{\alpha} n_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} [\mathbf{v}_0 \times \mathbf{B}] + \varepsilon_{\alpha} n_{\alpha} \mathbf{F}_{\alpha}^{(1)} \right] \right\} \\ & - \mathbf{v}_0 \cdot \sum_{\alpha} \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} + \sum_{\alpha} \tau_{\alpha}^{(0)} \mathbf{F}_{\alpha}^{(1)} \cdot \left[ \frac{\partial}{\partial t} (\rho_{\alpha} \mathbf{v}_0) + \frac{\partial}{\partial \mathbf{r}} \cdot \rho_{\alpha} \mathbf{v}_0 \mathbf{v}_0 + \frac{\partial}{\partial \mathbf{r}} \cdot p_{\alpha} \mathbf{I} - \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} - q_{\alpha} n_{\alpha} [\mathbf{v}_0 \times \mathbf{B}] \right] = 0. \end{aligned} \quad (2.6)$$

From Eq. (2.4) we have

$$\sum_{\alpha} \tau_{\alpha}^{(0)} \left[ \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} + \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha} \mathbf{v}_0 \times \mathbf{B} \right] = \mathbf{L}(t), \quad (2.7)$$

or

$$\sum_{\alpha} \tau_{\alpha}^{(0)} \left[ \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} + \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha} \mathbf{v}_0 \times \mathbf{B} \right] = \mathbf{L}, \quad (2.8)$$

where  $\mathbf{L}$  is constant vector. Let us introduce vector  $\mathbf{L}_{\alpha}(t)$

and rewrite now Eq. (2.5), which contains the density fluctuation [1]

$$\rho_{\alpha}^{\prime} = \tau_{\alpha}^{(0)} \left( \frac{\partial \rho_{\alpha}}{\partial t} + \frac{\partial}{\partial \mathbf{r}} \cdot (\rho_{\alpha} \mathbf{v}_0) \right). \quad (2.10)$$

We have

$$\begin{aligned} & \frac{\partial}{\partial t} \mathbf{L}(t) - \sum_{\alpha} \mathbf{F}_{\alpha}^{(1)} \rho_{\alpha}^a - \sum_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} \mathbf{L}_{\alpha}(t) \times \mathbf{B} \\ & + \mathbf{v}_0 \cdot \left\{ \frac{\partial}{\partial t} \cdot \sum_{\alpha} \tau_{\alpha}^{(0)} \left[ \mathbf{F}_{\alpha}^{(1)} \rho_{\alpha} + \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha} [\mathbf{v}_0 \times \mathbf{B}] \right] \right\} + \left( \sum_{\alpha} \tau_{\alpha}^{(0)} \left[ \mathbf{F}_{\alpha}^{(1)} \rho_{\alpha} + \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha} [\mathbf{v}_0 \times \mathbf{B}] \right] \cdot \frac{\partial}{\partial \mathbf{r}} \right) \mathbf{v}_0 \\ & + \sum_{\alpha} \tau_{\alpha}^{(0)} \left[ \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} + \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha} [\mathbf{v}_0 \times \mathbf{B}] \right] \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{v}_0 + \left( \mathbf{v}_0 \cdot \frac{\partial}{\partial \mathbf{r}} \right) \left\{ \sum_{\alpha} \tau_{\alpha}^{(0)} \left[ \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} + \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha} [\mathbf{v}_0 \times \mathbf{B}] \right] \right\} = 0, \end{aligned} \quad (2.11)$$

where

$$\rho_{\alpha}^a = \rho_{\alpha} - \rho_{\alpha}^{\prime}. \quad (2.12)$$

Using also (2.9), we find

$$\sum_{\alpha} \mathbf{F}_{\alpha}^{(1)} \rho_{\alpha}^a = \frac{\partial}{\partial t} \mathbf{L}(t) - \sum_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} \mathbf{L}_{\alpha}(t) \times \mathbf{B} + \left( \mathbf{L}(t) \cdot \frac{\partial}{\partial \mathbf{r}} \right) \mathbf{v}_0 + \mathbf{L}(t) \left( \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{v}_0 \right) + \left( \mathbf{v}_0 \cdot \frac{\partial}{\partial \mathbf{r}} \right) \mathbf{L}(t). \quad (2.13)$$

The vector product in Eq. (2.13) can be transformed as

$$\mathbf{L}_{\alpha}(t) \times \mathbf{B} = \tau_{\alpha}^{(0)} \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} \times \mathbf{B} - \tau_{\alpha}^{(0)} q_{\alpha} n_{\alpha} [\mathbf{v}_0 B^2 - \mathbf{B}(\mathbf{v}_0 \cdot \mathbf{B})] \quad (2.14)$$

where  $q_{\alpha} n_{\alpha}$  is the charge of  $\alpha$ -species in the unit volume.

Taking into account the relations (2.10), (2.11), (2.12), we can realize the analogical transformation of the energy condition (2.6):

$$\begin{aligned}
\mathbf{v}_0 \cdot \sum_{\alpha} \mathbf{F}_{\alpha}^{(1)} \rho_{\alpha}^a &= \frac{\partial}{\partial t} \sum_{\alpha} \tau_{\alpha}^{(0)} (\rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} \cdot \mathbf{v}_0) + \sum_{\alpha} \tau_{\alpha}^{(0)} \mathbf{F}_{\alpha}^{(1)} \cdot \left[ \rho_{\alpha} \frac{\partial \mathbf{v}_0}{\partial t} + \rho_{\alpha} \left( \mathbf{v}_0 \cdot \frac{\partial}{\partial \mathbf{r}} \right) \mathbf{v}_0 + \frac{\partial}{\partial \mathbf{r}} \cdot p_{\alpha} \tilde{\mathbf{I}} \right] \\
&+ \frac{\partial}{\partial \mathbf{r}} \cdot \left\{ \sum_{\alpha} \tau_{\alpha}^{(0)} \left[ \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} \cdot \mathbf{v}_0 \mathbf{v}_0 + p_{\alpha} \mathbf{F}_{\alpha}^{(1)} \cdot \tilde{\mathbf{I}} + \frac{1}{2} \rho_{\alpha} v_0^2 \mathbf{F}_{\alpha}^{(1)} + \frac{3}{2} \mathbf{F}_{\alpha}^{(1)} p_{\alpha} + \frac{\rho_{\alpha} v_0^2}{2} \frac{q_{\alpha}}{m_{\alpha}} [\mathbf{v}_0 \times \mathbf{B}] + \right. \right. \\
&\left. \left. + \frac{5}{2} p_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} [\mathbf{v}_0 \times \mathbf{B}] \right] \right\} \\
&+ \frac{\partial}{\partial \mathbf{r}} \cdot \sum_{\alpha} \varepsilon_{\alpha} n_{\alpha} \mathbf{L}_{\alpha} - \sum_{\alpha} \tau_{\alpha}^{(0)} \mathbf{F}_{\alpha}^{(1)} \cdot \mathbf{L}_{\alpha}
\end{aligned} \tag{2.15}$$

Equations (2.7), (2.13) and (2.15) define the system of the sufficient conditions for levitation.

The choice of the non-local parameter needs in the special consideration [3, 4]. The system of equations (2.1) – (2.3) convert in the system of quantum hydrodynamic equations by the suitable choice of the non-local parameter  $\tau$ . The relation between  $\tau$  and kinetic energy [3, 4] is used in quantum hydrodynamics

$$\tau = H / \mu u^2, \tag{2.16}$$

where  $u$  is the particle velocity,  $H$  is the coefficient of proportionality which reflects the state of the physical system. In the simplest case  $H$  is equal to the Plank constant  $\hbar$  and the corresponding relation (2.16) correlates with the Heisenberg inequality. From the first glance the approximation (2.16) is distinguished radically from the kinetic relation known from the theory of the rarefied gases

$$\tau = \Pi \frac{\nu \rho}{p}, \tag{2.17}$$

which is used for the calculation of the non-local parameter in the macroscopic hydrodynamic case ( $\nu$  is the kinematic viscosity). But it is not a case. In quantum approximation the value  $\nu^{qu} = \hbar / m$  has the dimension  $[cm^2/s]$  and can be called as quantum viscosity, for the electron species  $\nu^{qu} = \hbar / m_e = 1.577 \text{ cm}^2/s$ . If we take into account that the value  $p/\rho \sim \overline{V}^2$ , then the interrelation of (2.16) and (2.17) becomes obvious.

### 3. Some Particular Cases of the Levitation Conditions

Write down the system of the sufficient levitation conditions for the quasi-stationary case neglecting dissipation and the space derivatives in Eq. (2.13). We find

$$\sum_{\alpha} \mathbf{F}_{\alpha}^{(1)} \rho_{\alpha}^a = - \sum_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} \mathbf{L}_{\alpha}(t) \times \mathbf{B}. \tag{3.1}$$

$$\sum_{\alpha} \mathbf{F}_{\alpha}^{(1)} \rho_{\alpha}^a = \mathbf{B} \times \sum_{\alpha} \tau_{\alpha}^{(0)} \frac{q_{\alpha}}{m_{\alpha}} \left[ \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} + \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha} \mathbf{v}_0 \times \mathbf{B} \right]. \tag{3.2}$$

Introducing the current density

$$\mathbf{j}_{\alpha} = \tau_{\alpha}^{(0)} q_{\alpha} n_{\alpha} \mathbf{v}_0, \tag{3.3}$$

one obtains

$$\sum_{\alpha} \mathbf{F}_{\alpha}^{(1)} \rho_{\alpha}^a = \mathbf{B} \times \sum_{\alpha} \tau_{\alpha}^{(0)} q_{\alpha} n_{\alpha} \mathbf{F}_{\alpha}^{(1)} + \mathbf{B} \times \sum_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} [\mathbf{j}_{\alpha} \times \mathbf{B}]. \tag{3.4}$$

The right-hand-side of Eq. (3.4) contains the cross terms for the forces of the mass and electro- magneto-dynamic origin. The last term in Eq. (3.4) can be written also in the form

$$\mathbf{B} \times \sum_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} [\mathbf{j}_{\alpha} \times \mathbf{B}] = \sum_{\alpha} \rho_{\alpha} \tau_{\alpha}^{(0)} \left[ \frac{q_{\alpha}}{m_{\alpha}} \right]^2 \{ \mathbf{v}_0 B^2 - \mathbf{B}(\mathbf{v}_0 \cdot \mathbf{B}) \}. \tag{3.5}$$

Taking into account (2.16), (2.17) it is naturally to suppose that

$$k_B T \tau_{\alpha}^{(0)} \geq \hbar, \tag{3.6}$$

Introduce now

$$\tau_{\alpha}^{(0)} = A \frac{\hbar}{k_B T}, \tag{3.7}$$

where  $A$  is a parameter which leads to appearance the effective temperature  $T_{eff}$ . Other approximations can be used, for example

$$\tau_{\alpha}^{(0)} = \frac{\hbar}{k_B T_{\alpha, eff}}. \tag{3.8}$$

Let us consider now other particular case when  $\mathbf{v}_0 = 0$ . In equations (2.7), (2.13) and (2.15) we conserve the terms up to the  $\tau_{\alpha}^{(0)}$  order. From Eq. (2.13) follows

$$\sum_{\alpha} \mathbf{F}_{\alpha}^{(1)} \rho_{\alpha}^a = \frac{\partial}{\partial t} \mathbf{L}(t) - \sum_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} \mathbf{L}_{\alpha}(t) \times \mathbf{B}, \tag{3.9}$$

where now

$$\mathbf{L}_{\alpha}(t) = \tau_{\alpha}^{(0)} \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)}, \tag{3.10}$$

$$\mathbf{L}(t) = \sum_{\alpha} \tau_{\alpha}^{(0)} \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)}. \tag{3.11}$$

Then

in Eq. (3.9)

$$\sum_{\alpha} \mathbf{F}_{\alpha}^{(1)} \rho_{\alpha}^a = \frac{\partial}{\partial t} \sum_{\alpha} \tau_{\alpha}^{(0)} \rho_{\alpha}^a \mathbf{F}_{\alpha}^{(1)} - \sum_{\alpha} q_{\alpha} n_{\alpha}^a \tau_{\alpha}^{(0)} \mathbf{F}_{\alpha}^{(1)} \times \mathbf{B}. \quad (3.12)$$

$$\sum_{\alpha} \mathbf{F}_{\alpha}^{(1)} \rho_{\alpha}^a = \frac{\partial}{\partial t} \sum_{\alpha} \tau_{\alpha}^{(0)} \rho_{\alpha}^a \left( \mathbf{g} + \frac{q_{\alpha}}{m_{\alpha}} \mathbf{E} \right) - \sum_{\alpha} q_{\alpha} n_{\alpha}^a \tau_{\alpha}^{(0)} \left( \mathbf{g} + \frac{q_{\alpha}}{m_{\alpha}} \mathbf{E} \right) \times \mathbf{B} \quad (3.14)$$

Introduce the explicit expression for the mass force

where  $n_{\alpha}^a = \rho_{\alpha}^a / m_{\alpha}$ . From Eq. (3.14) follows

$$\mathbf{F}_{\alpha}^{(1)} = \mathbf{g} + \frac{q_{\alpha}}{m_{\alpha}} \mathbf{E} \quad (3.13)$$

$$\sum_{\alpha} \mathbf{F}_{\alpha}^{(1)} \rho_{\alpha}^a = \frac{\partial}{\partial t} \sum_{\alpha} \tau_{\alpha}^{(0)} \rho_{\alpha}^a \mathbf{g} + \frac{\partial}{\partial t} \left[ \mathbf{E} \sum_{\alpha} \tau_{\alpha}^{(0)} n_{\alpha}^a q_{\alpha} \right] - \mathbf{g} \times \mathbf{B} \sum_{\alpha} q_{\alpha} n_{\alpha}^a \tau_{\alpha}^{(0)} - \mathbf{E} \times \mathbf{B} \sum_{\alpha} \left( \frac{q_{\alpha}}{m_{\alpha}} \right)^2 \rho_{\alpha}^a \tau_{\alpha}^{(0)}. \quad (3.15)$$

Let us introduce in Eq. (3.15) the Umov – Pointing vector  $\mathbf{S}$  and Alexeev vector  $\mathbf{S}_A$  in the forms

$$\mathbf{S} = \mathbf{E} \times \mathbf{B}, \quad (3.16)$$

$$\mathbf{S}_A = \mathbf{g} \times \mathbf{B}. \quad (3.17)$$

In this case

$$\sum_{\alpha} \mathbf{F}_{\alpha}^{(1)} \rho_{\alpha}^a = \frac{\partial}{\partial t} \left( \sum_{\alpha} \tau_{\alpha}^{(0)} \rho_{\alpha}^a \mathbf{g} \right) + \frac{\partial}{\partial t} \left[ \mathbf{E} \sum_{\alpha} \tau_{\alpha}^{(0)} n_{\alpha}^a q_{\alpha} \right] - \mathbf{S}_A \sum_{\alpha} q_{\alpha} n_{\alpha}^a \tau_{\alpha}^{(0)} - \mathbf{S} \sum_{\alpha} \left( \frac{q_{\alpha}}{m_{\alpha}} \right)^2 \rho_{\alpha}^a \tau_{\alpha}^{(0)}. \quad (3.18)$$

For the approximation (3.7) one obtains

$$\sum_{\alpha} \mathbf{F}_{\alpha}^{(1)} \rho_{\alpha}^a = A \frac{\hbar}{k_B} \left[ \frac{\partial}{\partial t} \left( \frac{\rho^a}{T} \mathbf{g} \right) + \frac{\partial}{\partial t} \left( \mathbf{E} \frac{Q^a}{T} \right) - \mathbf{S}_A \frac{Q^a}{T} - \mathbf{S} \frac{1}{T} \sum_{\alpha} \left( \frac{q_{\alpha}}{m_{\alpha}} \right)^2 \rho_{\alpha}^a \right], \quad (3.19)$$

where the average charge density is introduced

$$Q^a = \sum_{\alpha} q_{\alpha} n_{\alpha}^a. \quad (3.20)$$

The analogical transformations of the energy condition (2.15) can be realized for this particular case when  $\mathbf{v}_0^a = 0$ . Namely

$$\begin{aligned} - \sum_{\alpha} \tau_{\alpha}^{(0)} \rho_{\alpha}^a \mathbf{F}_{\alpha}^{(1)} \cdot \sum_{\alpha} \mathbf{F}_{\alpha}^{(1)} \rho_{\alpha}^a &= \sum_{\alpha} \tau_{\alpha}^{(0)} \mathbf{F}_{\alpha}^{(1)} \cdot \left[ \frac{\partial}{\partial t} \cdot p_{\alpha}^a \tilde{\mathbf{I}} \right] + \frac{\partial}{\partial t} \cdot \left\{ \sum_{\alpha} \tau_{\alpha}^{(0)} \left[ p_{\alpha}^a \mathbf{F}_{\alpha}^{(1)} \cdot \tilde{\mathbf{I}} + \frac{3}{2} \mathbf{F}_{\alpha}^{(1)} p_{\alpha}^a \right] \right\} \\ &+ \frac{\partial}{\partial t} \cdot \sum_{\alpha} \varepsilon_{\alpha} n_{\alpha}^a \tau_{\alpha}^{(0)} \mathbf{F}_{\alpha}^{(1)}. \end{aligned} \quad (3.21)$$

For the approximation (3.7) we find from Eq. (3.21)

$$\begin{aligned} \frac{\partial}{\partial t} \cdot \sum_{\alpha} \left[ \frac{5}{2} p_{\alpha}^a + \varepsilon_{\alpha} n_{\alpha}^a \right] \mathbf{F}_{\alpha}^{(1)} - \sum_{\alpha} \left( \frac{5}{2} p_{\alpha}^a + \varepsilon_{\alpha} n_{\alpha}^a \right) \mathbf{F}_{\alpha}^{(1)} \cdot \frac{\partial \ln T}{\partial t} \\ = - \left[ \sum_{\alpha} \rho_{\alpha}^a \mathbf{F}_{\alpha}^{(1)} \right]^2 - \sum_{\alpha} \mathbf{F}_{\alpha}^{(1)} \cdot \frac{\partial p_{\alpha}^a}{\partial t}. \end{aligned} \quad (3.22)$$

Eq. (3.22) should be considered as a relation defining the energy consumption needed for the levitation.

From (3.13) follows a relation

$$\sum_{\alpha} \mathbf{F}_{\alpha}^{(1)} \rho_{\alpha}^a = \mathbf{g} \rho^a + \mathbf{E} \sum_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha}^a = \mathbf{g} \rho^a + \mathbf{E} \sum_{\alpha} q_{\alpha} n_{\alpha}^a = \mathbf{g} \rho^a + \mathbf{E} Q^a \quad (3.23)$$

which can be used for the transformation of Eq. (3.22). For a tentative estimate we can omit the derivatives of the logarithmic terms and the time derivatives for a quasi-neutral

media. As a result from (3.22)

$$\begin{aligned} \frac{\partial}{\partial t} \cdot \left\{ \mathbf{g} \sum_{\alpha} \left[ \frac{5}{2} p_{\alpha}^a + \varepsilon_{\alpha} n_{\alpha}^a \right] \right\} + \frac{\partial}{\partial t} \cdot \left\{ \mathbf{E} \sum_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} \left[ \frac{5}{2} p_{\alpha}^a + \varepsilon_{\alpha} n_{\alpha}^a \right] \right\} \\ = - \left[ \rho^a \mathbf{g} + \mathbf{E} Q^a \right]^2 - \mathbf{g} \cdot \frac{\partial \rho^a}{\partial t} - \mathbf{E} \cdot \sum_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} \frac{\partial p_{\alpha}^a}{\partial t}. \end{aligned} \quad (3.24)$$

For a quasi-neutral media

$$Q^a = 0, \quad (3.25)$$

then

$$\begin{aligned} \frac{\partial}{\partial t} \cdot \left[ \mathbf{g} \left( \frac{5}{2} p^a + \Xi \right) \right] + \frac{\partial}{\partial t} \cdot \left\{ \mathbf{E} \sum_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} \left[ \frac{5}{2} p_{\alpha}^a + \Xi_{\alpha} \right] \right\} \\ = - \left[ \rho^a \mathbf{g} \right]^2 - \mathbf{E} \cdot \sum_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} \frac{\partial p_{\alpha}^a}{\partial t} - \mathbf{g} \cdot \frac{\partial \rho^a}{\partial t}, \end{aligned} \quad (3.26)$$

where

$$\Xi = \sum_{\alpha} \varepsilon_{\alpha} n_{\alpha}^a, \quad \Xi_{\alpha} = \varepsilon_{\alpha} n_{\alpha}^a. \quad (3.27)$$

Let us obtain a tentative estimate from (3.19) for the quasi-stationary case in a quasi-neutral media. From (3.18) for the case under consideration we have

$$\sum_{\alpha} \mathbf{F}_{\alpha}^{(1)} \rho_{\alpha}^a = -\mathbf{S}_A \sum_{\alpha} q_{\alpha} n_{\alpha}^a \tau_{\alpha}^{(0)} - \mathbf{S} \sum_{\alpha} \left( \frac{q_{\alpha}}{m_{\alpha}} \right)^2 \rho_{\alpha}^a \tau_{\alpha}^{(0)}. \quad (3.28)$$

From (3.23), (3.28) we find

$$\rho^a \mathbf{g} = -\mathbf{S}_A \sum_{\alpha} q_{\alpha} n_{\alpha}^a \tau_{\alpha}^{(0)} - \mathbf{S} \sum_{\alpha} \left( \frac{q_{\alpha}}{m_{\alpha}} \right)^2 \rho_{\alpha}^a \tau_{\alpha}^{(0)}, \quad (3.29)$$

or

$$\rho^a \mathbf{g} = -A \frac{\hbar}{k_B} \mathbf{S} \frac{1}{T} \sum_{\alpha} \left( \frac{q_{\alpha}}{m_{\alpha}} \right)^2 \rho_{\alpha}^a \quad (3.30)$$

in the case of (3.7) approximation. Relation leads in SI to the estimate

$$\rho^a \mathbf{g} \cong -A \frac{\mathbf{S}}{T} \cdot 2.138 \cdot 10^{-19} n_e^a \quad (3.31)$$

or

$$\rho^a \mathbf{g} \cong 2.138 \cdot 10^{-19} \frac{A}{T} [\mathbf{B} \times \mathbf{E}] n_e^a. \quad (3.32)$$

The following Table 1 lists known examples of number densities at 1 atm and 20 °C, unless otherwise noted.

**Table 1.** Molecular number density and related parameters of some materials

Material	Number density ( $n$ )	Density ( $\rho$ )
Units	$(10^{27} \text{ m}^{-3})$ or $(10^{21} \text{ cm}^{-3})$	$(10^3 \text{ kg/m}^3)$ or $(\text{g/cm}^3)$
dry air	0.02504	$1.2041 \times 10^{-3}$
water	33.3679	0.9982
diamond	176.2	3.513

Let us use now (1.4) and the obvious phenomenological condition of the force balance (see also [9]) we have

$$\mathbf{F} = \frac{\chi}{\mu_0 \mu} \text{grad} B^2 = \rho \mathbf{g} \hat{\mathbf{e}}_z, \quad (3.33)$$

where  $\rho$  is the mass density of the material to be levitated and  $\hat{\mathbf{e}}_z$  is the unit vector in the vertical direction, magnetic susceptibility  $\chi$  is negative for diamagnetic materials. In the frame of the phenomenological description of the magnetic and gravitational field we have

$$w = w_m + w_g = -\frac{\chi}{\mu \mu_0} B^2 + \rho g z. \quad (3.34)$$

A necessary condition for stability is

$$\int_S \mathbf{F} \cdot d\mathbf{s} < 0, \quad (3.35)$$

where  $S$  is any small closed surface surrounding the equilibrium point. It leads to the condition

$$\text{div } \mathbf{F} < 0. \quad (3.36)$$

This relation leads to the stability condition

$$\Delta w = \text{divgrad } w = \text{divgrad } w_m = -\frac{\chi}{\mu \mu_0} \Delta B^2 = -\text{div} \mathbf{F} > 0 \quad (3.37)$$

if  $\chi < 0$  (diamagnetic materials) and  $\Delta B^2 > 0$ . The corresponding stability investigation from the phenomenological point of view was realized in [6].

From the relation (3.33) follows ( $\mu \sim 1$ )

$$\rho \mathbf{g} = 2 \frac{\chi}{\mu_0} B \frac{\partial B}{\partial z} \quad (3.38)$$

and from (3.29)

$$\rho^a \mathbf{g} = -\mathbf{S} \tau^{(0)} \sum_{\alpha} \left( \frac{q_{\alpha}}{m_{\alpha}} \right)^2 \rho_{\alpha}^a, \quad (3.39)$$

if the non-local parameter does not depend on the sort of species  $\alpha$ . After equalizing the right-hand-sides of relations (3.38) and (3.39) one obtains

$$\tau^{(0)} \mu_0 E \frac{q_e}{2m_e} q_e n_e^a = \chi \frac{\partial B}{\partial z}, \quad (3.40)$$

because

$$\sum_{\alpha} \left( \frac{q_{\alpha}}{m_{\alpha}} \right)^2 \rho_{\alpha}^a \cong \left( \frac{q_i}{m_i} \right)^2 \rho^a + \left( \frac{q_e}{m_e} \right)^2 \rho_e^a = \frac{q_i^2}{m_i} n_i^a + \frac{q_e^2}{m_e} n_e^a \cong \left( \frac{q_e}{m_e} \right)^2 \rho_e^a \quad (3.41)$$

Let us introduce the character length  $l_m$

$$l_m = \mu_0 \frac{q_e^2}{2m_e}, \quad (3.42)$$

hence from (3.40), (3.42)

$$l_m E n_e^a = -|\chi| \frac{1}{\tau^{(0)}} \frac{\partial B}{\partial z}. \quad (3.43)$$

Introduce the electromotive force (EMF) for a particle

$$E_{ind} = l_m E, \quad (3.44)$$

and for  $n_e^a$  particles

$$E_{ind,n} = l_m E n_e^a. \quad (3.45)$$

Hence from (3.43), (3.45) we find

$$E_{ind,n} = -\frac{|\chi|}{\tau^{(0)}} \frac{\partial B}{\partial z}. \quad (3.46)$$

Formally Eq. (3.46) can be written in the form of Faraday's law of induction, the most widespread version of this law states that the induced electromotive force in any closed circuit is equal to the rate of change of the magnetic flux through the circuit:

$$E_{ind} = -\frac{\partial \Phi_B}{\partial t}, \quad (3.47)$$

where  $\Phi_B$  is the magnetic flux. This version of Faraday's law strictly holds only when the closed circuit is a loop of infinitely thin wire and is invalid in some other circumstances. Nevertheless formally

$$E_{ind,n} = -|\chi| \frac{1}{\tau^{(0)}} \frac{\partial B}{\partial t} \frac{\partial t}{\partial z}, \quad (3.48)$$

or

$$E_{ind,n} = -|\chi| \frac{1}{\tau^{(0)}} \frac{\partial B}{\partial t} \frac{1}{v_m}. \quad (3.49)$$

After introduction of the character counter square

$$S_m = \frac{|\chi|}{\tau^{(0)} v_m n_e^a}, \quad (3.50)$$

we reach the relation in the form of Faraday's law

$$E_{ind} = -\frac{\partial \Phi_B}{\partial t}. \quad (3.51)$$

## 4. Conclusion

The following conclusions of the principal significance can be done:

1. The levitation effects are the direct consequence of the non-local equations (2.1) – (2.3).
2. The sufficient conditions of levitation are the particular case of Eqs. (2.1) – (2.3).
3. The strict theory of levitation can be constructed only in the frame of non-local physics.

4. Fluctuations of the gravitational field lead to the electro-magneto dynamical fluctuations and verse versa. This fact can effect on the work of electronic devices during the evolution of the wave atmospheric fronts.

5. Levitation effects are connected not only with the electro-magnetic energy flux  $S$ , but also with the cross flux  $S_A$ .

6. Usual local conditions of levitation are the deep particular cases of the non-local theory.

## References

- [1] Alexeev, B.V. The Generalized Boltzmann Equation, Generalized Hydrodynamic Equations and their Applications. *Phil. Trans. Roy. Soc. Lond.* 349 p.417-443 (1994)
- [2] Alexeev, B.V. Generalized Boltzmann physical kinetics. – Amsterdam: Elsevier, 2004. 376 p.
- [3] Alexeev, B.V. Generalized quantum hydrodynamics and principles of non-local physics. *J. of Nanoelectronics and Optoelectronics*. (2008) No. 3. P. 143–158.
- [4] Alexeev, B.V. Application of Generalized Quantum Hydrodynamics in the Theory of Quantum Soliton Evolution, *J. Nanoelectron. Optoelectron.* (2008) No. 3, p. 316 -328.
- [5] Earnshaw, S., On the nature of the molecular forces which regulate the constitution of the luminiferous ether, *Trans. Cambridge Phil. Soc.* 7, 97-112, 1842.
- [6] Berry, M.V. and Geim, A.K., “Of flying frogs and levitrons”, *Euro. J. Phys.*, 18, 307–313 (1997) and <http://www.hfml.sci.kun.nl/hfml/levitate.html>.
- [7] Geim, A.K., “Everyone's Magnetism”, *Phys. Today*, 51 Sept., 36–39 (1998).
- [8] Geim, A. K, M. Simon, M.D., Boamfa, M.I. and Heflinger, O.L., “Magnet levitation at your fingertips”, *Nature*, 400, 323–324 (1999).
- [9] Simon, M.D, Geim, A.K. “Diamagnetic levitation; Flying frogs and floating magnets”, *J. App. Phys.*, 87, 6200–6204 (2000).
- [10] Berry, M.V. The LEVITRON and adiabatic trap for spins, *Proc. Roy Soc. Lond., A* (1996) 452, 1207-1220.
- [11] Harrigan, R.M., Levitation device, U.S. Patent \$,382245, May 3, 1983.